Sachdev–Ye–Kitaev model: from statistical mechanics to anti-de Sitter and de Sitter holographies

Alexey Milekhin (Caltech IQIM)

Oct 25, 2024

Cosmology and High Energy Physics workshop, Montpellier University

Motivation

- Large complicated systems can exhibit universal behavior

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- Classical thermodynamics
- Statistics of energy level gaps of heavy nuclei (and chaotic systems) can be described by a random Hamiltonian matrix

Wigner; Dyson 1950s

- With the advent of quantum mechanics and quantum field theory we can look at more microscopic examples.

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$$H = \sum_{i,j=1}^{N} J_{ij} \psi_i \psi_j$$

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- In the limit of large N many physical quantities have small fluctuations: enough to consider one sample of J_{ij} (self-averaging)
- Let us consider interacting fermions:

$$H_{SYK} = \sum_{i,j,k,l=1}^{N} J_{ijkl}\psi_i\psi_j\psi_k\psi_l$$

VALIDITY OF RANDOM MATRIX THEORIES FOR MANY-PARTICLE SYSTEMS*

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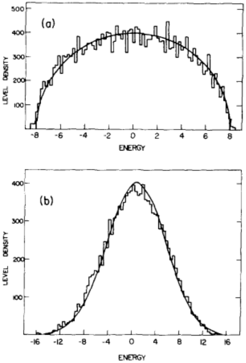

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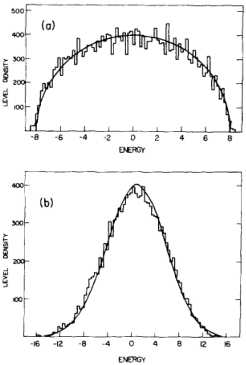

Historical perspective:

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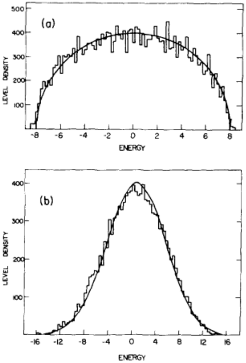

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1973 – Quantum chromodynamics of quark and gluons

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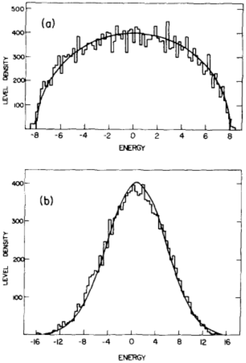
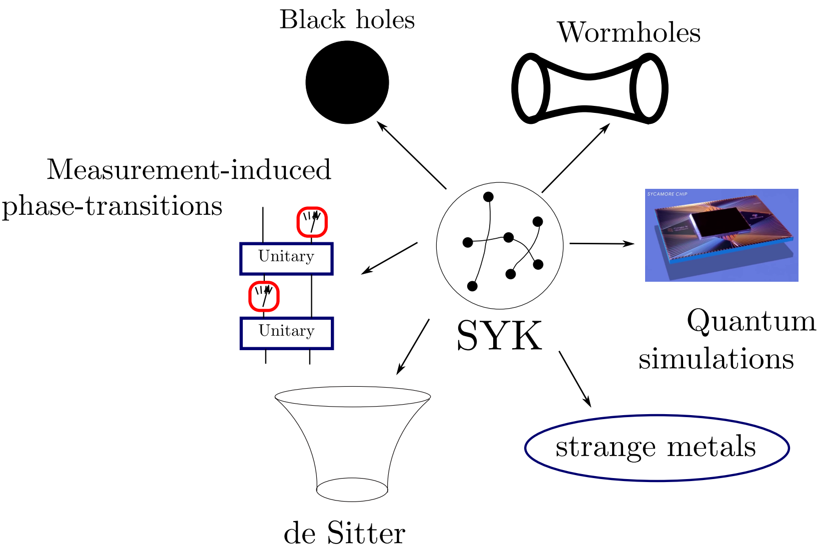


Historical perspective:
1973 – Quantum chromodynamics of quark and gluons
1969 – de Gaulle is still president

A lot of results in the past 9 years from diverse research communities



Plan

- Overview of SYK: large N limit and reparametrizations at finite temperature
- Applications: black holes and wormholes in anti-de Sitter
- WIP: From SYK to higher-dimensional de Sitter

  AM–Narovlansky–Verlinde–Xu

N Majorana fermions $\psi_i, i = 1, \ldots, N$:

$$\{\psi_i, \psi_j\} = \delta_{ij} \to 2^{N/2} \times 2^{N/2} \text{ matrices}$$

SYK model

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$$\mathcal{L}_{SYK} = \frac{1}{2} \underbrace{\psi_i \partial_\tau \psi_i}_{G_0^{-1}} - \sum_{ijkl=1}^{N} J_{ijkl} \psi_i \psi_j \psi_k \psi_l, \quad \langle J_{ijkl}^2 \rangle = \frac{6J^2}{N^3} - \text{Gaussian}$$

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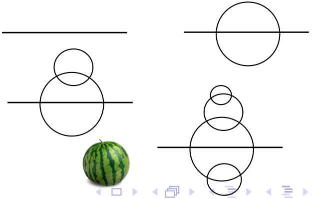
All-to-all interaction: no "space" only time
Large N Schwinger–Dyson equations for 2-point function
$G = \langle \psi_i(\tau) \psi_i(0) \rangle$:

Sachdev–Ye'1993 (complex fermions)

$$(-i\omega_n - \Sigma(\omega_n)) G(\omega_n) = 1$$

$$\Sigma(\tau) = J^2 G(\tau)^3$$

(Euclidean version)



Is disorder really necessary?

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No: there are tensor models with the same large N limit:

Gurau'2013

$$H_{CTKT} = J \sum_{abc,a'b'c'=1}^{N} \psi_{abc}\psi_{a'b'c}\psi_{ab'c'}\psi_{a'bc'}$$

Carroza–Tanasa'2015;Klebanov–Tarnopolsky'2016

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Klebanov–AM–Popov–Tarnopolsky'2018

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Possible to match even some finite N effects

Klebanov–AM–Popov–Tarnopolsky'2018

Price: huge $O(N)^3$ symmetry. Leads to interesting quantum-error correction effects in the singlet subspace (non-abelian stabilizer code)

AM'2020

At low temperatures one can neglect the kinetic term:

$$\cancel{\partial_\tau G} - J^2 \int d\tau' [G^3](\tau - \tau') \cdot G(\tau') = \delta(\tau)$$

Finite-temperature solution:

$$G \propto \frac{\operatorname{sgn}(\tau)}{\sqrt{J\beta \sin\left(\frac{\pi|\tau|}{\beta}\right)}}$$

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The smallness of kinetic term leads to emergent reparametrization symmetry:

$$G(\tau_1, \tau_2) \to \left(f(\tau_1)' f(\tau_2)'\right)^{1/4} G(f(\tau_1), f(\tau_2))$$

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For thermodynamics and higher-point functions this is too crude.

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$$S(f(\tau)) = -\frac{N\alpha_S}{J}\int d\tau \underbrace{\left(\frac{f'''}{f'} - \frac{3}{2}\left(\frac{f''}{f'}\right)^2\right)}_{\textit{Schwarzian}}$$

Kitaev'2015; Maldacena–Stanford'2016

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More generally, for coupled SYK models one has non-local action:

Maldacena–Stanford–Yang'2016

AM'2021

$$S(f(\tau)) = -\frac{N\alpha_h}{J}\int d\tau_1 d\tau_2 \left(\frac{f'(\tau_1)f'(\tau_2)}{(f(\tau_1)-f(\tau_2))^2}\right)^h$$

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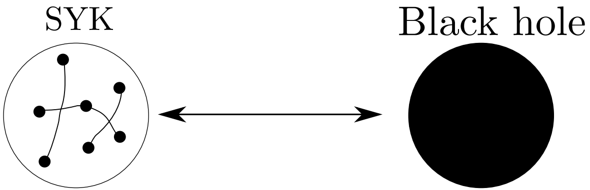
These actions describe the interesting physics: thermodynamics, transport, higher-point correlation functions

Application: Black holes and wormholes

SYK at finite temperature is like a black hole



- Reparametrizations suggest gravitational interpretation.

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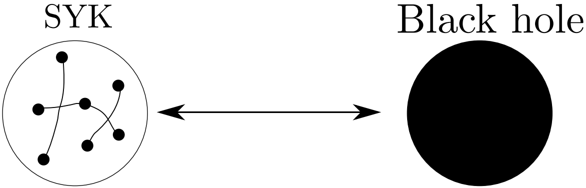

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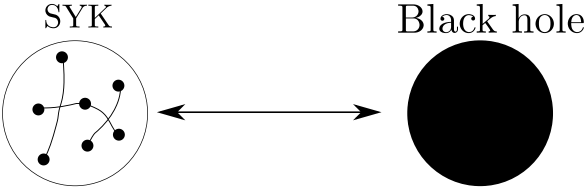

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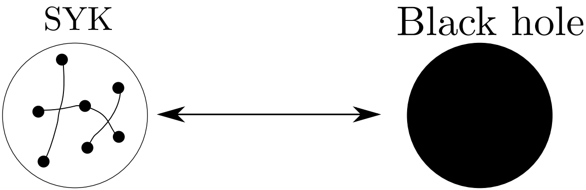

- Reparametrizations suggest gravitational interpretation.
- Conformal symmetry of AdS_2 (Poincare disk)
- Zero-temperature entropy $S_0 \approx 0.23N$ (like a charged black hole)
- Maximally chaotic, as measured by the out-of-time ordered correlator (OTOC): $\lambda_L = \frac{2\pi}{\beta}$

$$\langle\{\psi_i(t), \psi_j(0)\}^2\rangle = \frac{c}{N} e^{\lambda_L t} + \mathcal{O}\left(\frac{1}{N^2}\right)$$

Kitaev'2015

(butterfly effect)

- Schwarzian action does in fact govern Jackiw–Teitelboim (JT) gravity in AdS_2 (Poincare disk)

Maldacena–Stanford–Yang'2016

$$S_{JT} = \underbrace{\frac{\phi_0}{2}\left(\int R + 2\int_\partial K\right)}_{\text{topological}} + \frac{1}{2}\left(\underbrace{\int \phi(R+2)}_{\text{sets R=}-2} + \underbrace{2\phi_b \int_\partial K}_{\text{Sch origin}}\right)$$

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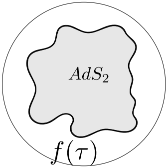
- $f(\tau)$ determines the boundary shape



- JT gravity arises as a certain limit of 4d Einstein gravity. Lessons about gravity around us:

Iliesiu, Turiaci

- SYK provides a UV-completion for a gravitational theory!!

How about wormholes?

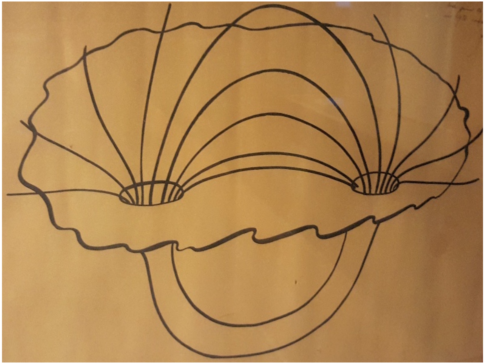
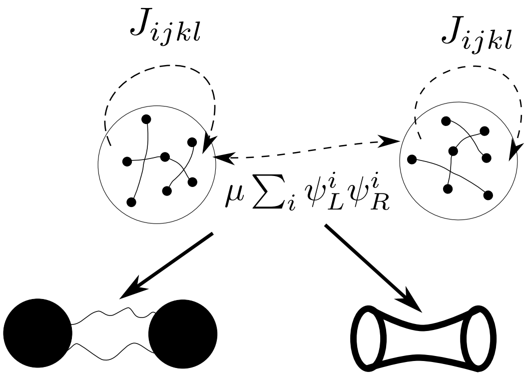


Figure: Drawing of a wormhole by John Wheeler, circa 1960s

For realistic traversable wormhole solution in 4d Einstein gravity:

Maldacena–AM–Popov'2018



There are indeed wormhole solutions:

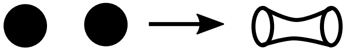
$$H = H_{\mathrm{SYK,L}} + H_{\mathrm{SYK,R}} + i\mu \sum_k \psi_L^k \psi_R^k$$

Maldacena–Qi'2018

μ mimics the exchange of quanta between the black holes

Wormhole formation

- Can we form a wormhole dynamically? It is not a smooth process



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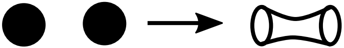
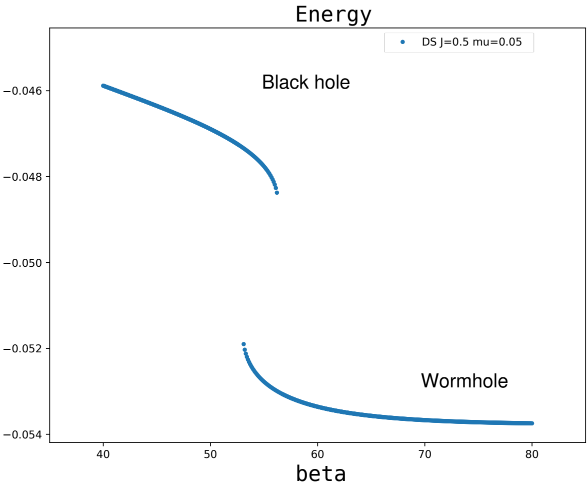


  to answer this questions we need a UV completion
- Cannot answer in Einstein gravity.

  Decay rate estimation: Bintanja–Freivogel–Rolph'2023
- Can answer in SYK!

Equilibrium thermodynamics: black hole and wormhole



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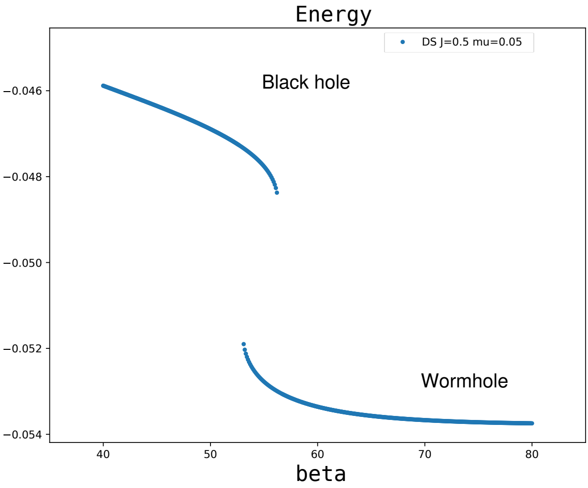
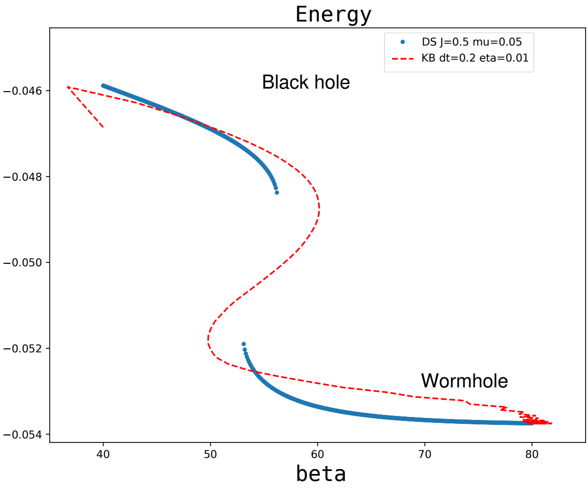


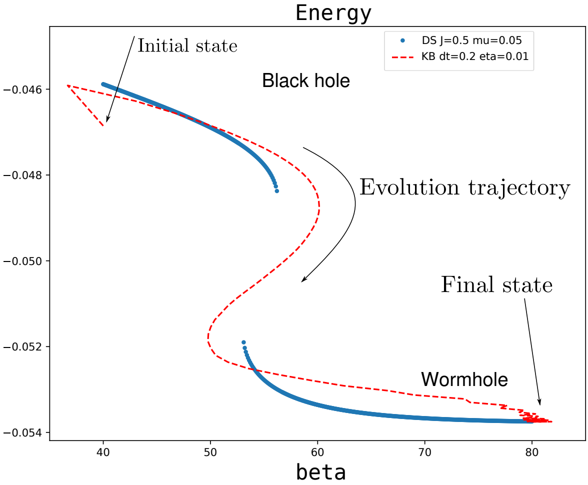
But can we switch between the two phases dynamically?

Maldacena–AM'2019



Forming a wormhole dynamically by cooling down black holes

Maldacena–AM'2019



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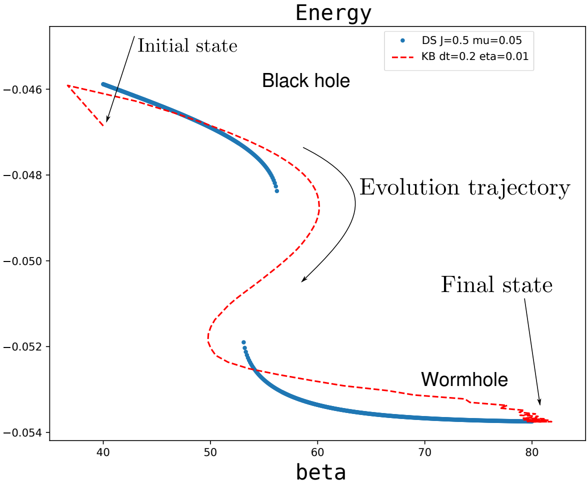
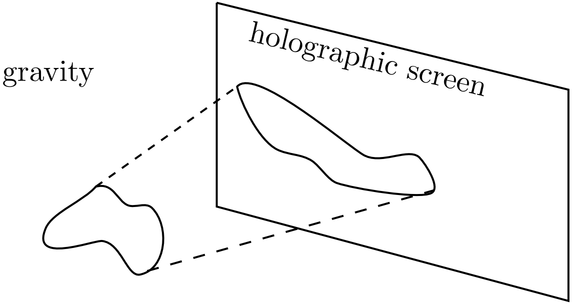


In 4d Einstein gravity the mechanism has to be different

Holographic principle and our Universe

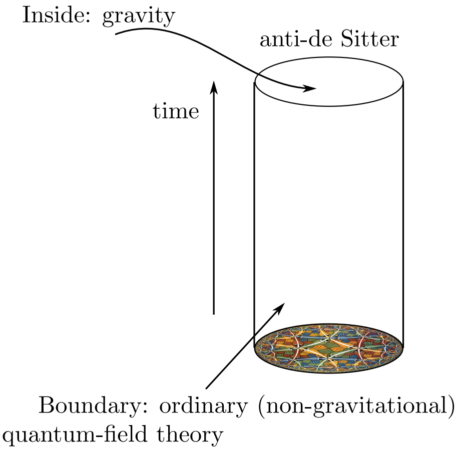
Gravitational theories are dual to quantum mechanical systems living on their boundary

't Hooft; Thorn; Susskind'1993



One concrete example: AdS/CFT

Maldacena'1997



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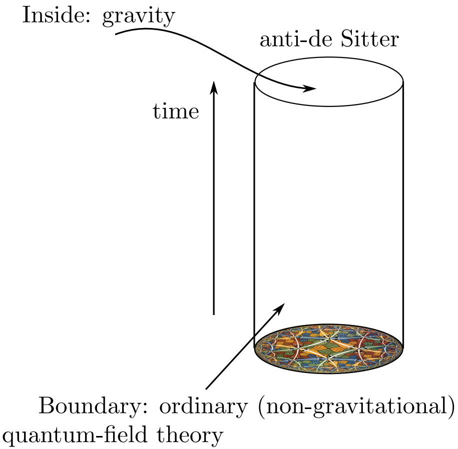

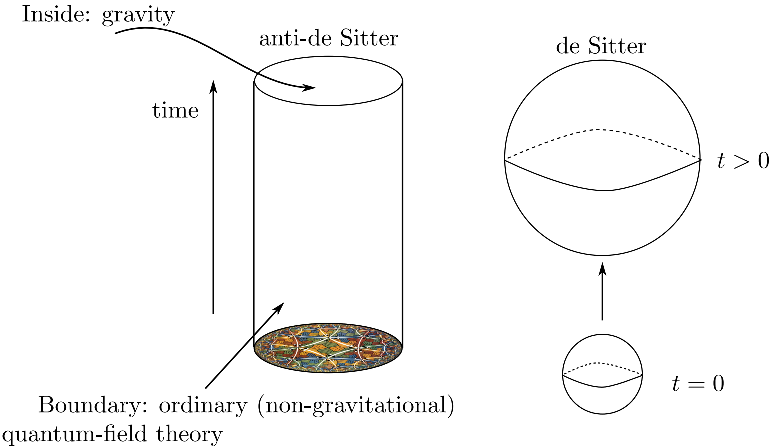

Our Universe is close to being de Sitter.

- There have been countless attempts to formulate holography for de Sitter space

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- In case of SYK, the relation to anti-de Sitter is not immediately obvious from the Hamiltonian

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- In case of SYK, the relation to anti-de Sitter is not immediately obvious from the Hamiltonian
- One has to compute 2-point function and notice it matches anti-de Sitter answer
- Can we reproduce de Sitter matter 2-point function from SYK?

Enter Narovlansky–Verlinde model

- Take two **non-interacting** SYK models

$$H = H_{\mathrm{SYK,L}} + H_{\mathrm{SYK,R}}$$

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$$H_{\mathrm{SYK,L}} - H_{\mathrm{SYK,R}} = 0$$

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as a **gauge constraint**

- Because of that, choose your physical operators carefully:

$$\mathcal{O}_{\mathrm{phys}}^{\Delta_L,\Delta_R}(t) = \int_{-\infty}^{+\infty} dt' \; \mathcal{O}_{\mathrm{L}}^{\Delta_L}(t-t')\mathcal{O}_{\mathrm{R}}^{\Delta_R}(t')$$

This reproduces correlation functions in de Sitter

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- $\Delta_L = \Delta, \Delta_R = 1-\Delta$:

$$\langle \mathcal{O}_{\mathrm{phys}}(t_1)\mathcal{O}_{\mathrm{phys}}(t_2)\rangle = \mathcal{N}_3 G_{\mathrm{dS}_3}(\underbrace{t_1 - t_2}_{\text{geodesic distance}})$$

Narovlansky–Verlinde'2023

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Narovlansky–Verlinde'2023

- $\Delta_L = \Delta, \Delta_R = (d-1)/2 - \Delta$:

$$\langle \mathcal{O}_{\text{phys}}(t_1)\mathcal{O}_{\text{phys}}(t_2)\rangle = \mathcal{N}_d \, G_{\text{dS}_d}(t_1 - t_2)$$

AM–Narovlansky–Verlinde–Xu'wip

A bizarre dictionary

- SYK at low temperature $\leftrightarrow$ dS Hartle–Hawking state (rescale time by β/π)
- For scalar fields: $\Delta_{SYK} = h/2$, $h(d-1-h) = m_{dS}^2$
- As usual, $G_N \sim 1/N$.
- Higher-point functions are to be matched... As well as deforming away from the Hartle–Hawking state

In closing: food for thought

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- A lot of physical systems, including some black holes, equilibrate diffusively. E.g. black hole in anti-de Sitter has quasi-normal frequencies:

  Policastro–Son–Starinets'2001

  $$\omega = -ip^2 D \to \partial_t \rho = D\Delta\rho$$

  (p - momentum along the horizon)

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  AM–Xu'2024

In closing: food for thought

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- An exotic phenomena in the realm of quantum many-body physics!

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- However JT gravity **with** matter is ill-defined even for simpliest geometries (e.g. cylinder). SYK is supposed to be dual to JT with matter. What can SYK say about it?

Thank you!

A bit about strange metals

Strange metals

- 1980s: discovery of high-temperature cuprate superconductors
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- Absence of quasiparticles, like SYK

In SYK Green function has no poles, only a branch-cut - non-Fermi liquid behavior:

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Song–Jian–Balents'2017

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Song–Jian–Balents'2017

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In principle, more general coupling can lead to other temperature powers:

AM'wip

$$1/\sigma_{elect} \propto T^{\alpha}/N$$

and arbitrary large ratio between heat- and electric conductivities (Lorentz ratio):

$$\frac{\sigma_{heat}}{\sigma_{elect}} \propto \frac{1}{T^{p}}$$