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# Non gaussianities from a gaussian theory

w/ H. Bergeron, J.-P. Gazeau, P. Małkiewicz – *Phys.Rev.* **D109/110** (2024)

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# Non gaussianities from a gaussian theory

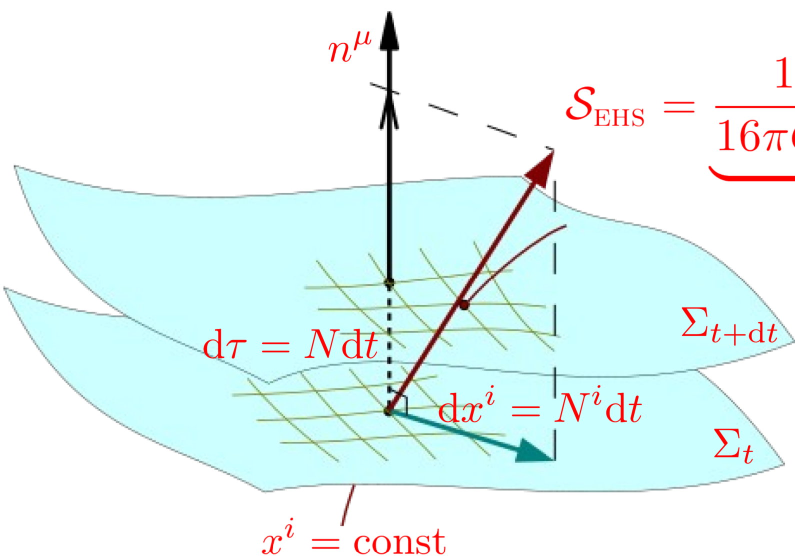
- **Enhanced quantization on the half line**
- **A new class of exact coherent states**

H. Bergeron, J.-P. Gazeau, P. Małkiewicz, PP – 2310.16868
*Phys.Rev.* **D109**, 023516 (2024)

- **Born-Oppenheimer, entanglement and the multiverse**

H. Bergeron, P. Małkiewicz, PP – 2405.09307
*Phys.Rev.* **D110**, 043512 (2024)

## Basic model: GR + perfect fluid




$$\mathcal{S}_{\text{EHS}} = \underbrace{\frac{1}{16\pi G_{\text{N}}}\int \mathrm{d}^4x\sqrt{-g}R}_{\mathcal{S}_{\text{EH}}} + \underbrace{\int \mathrm{d}^4x\sqrt{-g}P(w,\phi)}_{\mathcal{S}_{\text{M}}}$$

Canonical quantization
Wheeler De Witt

# Basic model: GR + perfect fluid

$$\mathcal{S}_{\rm EHS} = \underbrace{\frac{1}{16\pi G_{\rm N}}\int {\rm d}^4x\sqrt{-g}R}_{\mathcal{S}_{\rm EH}} + \underbrace{\int {\rm d}^4x\sqrt{-g}P(w,\phi)}_{\mathcal{S}_{\rm M}}$$

FLRW metric

$${\rm d}s^2 = -N^2(\tau){\rm d}\tau^2 + a^2(\tau)\gamma_{ij}{\rm d}x^i{\rm d}x^j$$

$$-\frac{1}{2\kappa}\int {\rm d}\tau N a^3 \underbrace{\int \sqrt{\gamma}{\rm d}^3x}_{\mathcal{V}_0}\underbrace{\frac{6\dot{a}^2}{a^2N^2}}_{R}$$

$$P = w\rho$$

$$N = (1+w)a^{3w} \implies H_{\rm M}^{(0)} = p_\tau$$

Use as clock

$$q = \frac{4\sqrt{6}}{3(1-w)\sqrt{1+w}}a^{\frac{3}{2}(1-w)} \equiv \gamma a^{\frac{3}{2}(1-w)}$$

$$p = \frac{\sqrt{6(1+w)}}{2\kappa_0}a^{\frac{3}{2}(1+w)}\frac{\dot{a}}{Na}$$

New canonical variables

$\frac{\dot{a}}{Na} \to H$

Hubble rate

$$H^{(0)} = 2\kappa_0 p^2 \;\to\; \kappa/\mathcal{V}_0 \;\xrightarrow{\kappa_0 \to \frac{1}{2}}\; H = p^2$$

# Coherent state quantization

Background phase space $(q,p) \in \mathbb{R}^{*+} \times \mathbb{R} = \{(q,p) | q > 0, p \in \mathbb{R}\}$

Natural choice = 2 parameter affine group of the real line $\lambda \in \mathbb{R} \mapsto (q,p) \cdot \lambda = \dfrac{\lambda}{q} + p$

$$\{(q_0,p_0)\,,(q,p)\} \mapsto (q',p') = (q_0,p_0) \circ (q,p) = \left(q'q, \frac{p}{q'} + p'\right)$$

left-invariant measure $\mathrm{d}q' \wedge \mathrm{d}p' = \mathrm{d}q \wedge \mathrm{d}p$

# "Usual" Coherent state quantization

Background phase space $(q,p) \in \mathbb{R}^{*+} \times \mathbb{R}$

Natural choice = 2 parameter affine group of the real line $\lambda \in \mathbb{R} \mapsto (q,p)\cdot\lambda = \frac{\lambda}{q} + p$

unitary, irreducible and square-integrable representation in the Hilbert space $\mathcal{H} = L^2\left(\mathbb{R}^{*+}, \mathrm{d}x\right)$

$$\langle x|U(q,p)|\psi\rangle = \langle x|q,p\rangle = \frac{\mathrm{e}^{ipx}}{\sqrt{q}}\psi\left(\frac{x}{q}\right)$$

$$\psi(x) = \langle x|\psi\rangle$$

covariant integral quantization on (affine) coherent states

$$\mathbb{R}^{+} \times \mathbb{R} \ni (q,p) \mapsto |q,p\rangle := U(q,p)|\xi\rangle \in \mathcal{H}$$

normalized fiducial state

$$U(q,p) = \mathrm{e}^{ip\hat{x}}\mathrm{e}^{-i(\ln q)\hat{d}}$$

algebra

$$[\hat{x},\hat{d}] = i\hat{x}$$

dilation $\hat{d} := \frac{1}{2}(\hat{x}\hat{p} + \hat{p}\hat{x})$

# **"Enhanced"** Coherent state quantization

Background phase space $(q,p) \in \mathbb{R}^{*+} \times \mathbb{R}$

Natural choice = 2 parameter affine group of the real line $\lambda \in \mathbb{R} \mapsto (q,p)\cdot\lambda = \frac{\lambda}{q} + p$

Equivalent representation $\Longrightarrow$ canonical transformation $q \mapsto q^\alpha \quad \& \quad p \mapsto \frac{p}{\alpha q^{\alpha-1}}$

unitary, irreducible and square-integrable representation in the Hilbert space $\mathcal{H}_\alpha = L^2\left(\mathbb{R}^{*+}, x^{-\alpha}\mathrm{d}x\right)$

covariant integral quantization on (affine) coherent states

$$\mathbb{R}^+ \times \mathbb{R} \ni (q,p) \mapsto |q,p\rangle := \hat{V}_{q,p}|\xi_0\rangle \in \mathcal{H}$$

normalized fiducial state

$$\hat{V}_{q,p} = e^{\frac{p}{2q}\hat{x}^2} e^{-(\ln q^2)\,\hat{d}/2} = e^{\frac{p}{2q}\hat{x}^2} e^{-(\ln q)\,\hat{d}}$$

algebra

$$\left[\hat{x}^\alpha, \frac{\hat{d}}{\alpha}\right] = i\hat{x}^\alpha$$

dilation $\hat{d} := \frac{1}{2}(\hat{x}\hat{p} + \hat{p}\hat{x})$

$\alpha \to 2$

$$\tilde{H} = p^2 + Cq^{-2}$$

$\left(\tilde{H},\ q^2,\ d/2\right)$ = closed Lie algebra with structure constant independent of $C$

define $c_\gamma(\psi) = \int_0^{+\infty} \frac{\mathrm{d}x}{x^{\gamma+2}} |\psi(x)|^2 \Longrightarrow \int_{\Pi_+} \frac{\mathrm{d}q\mathrm{d}p}{2\pi c_0(\xi_0)} |q,p\rangle\langle q,p| = 1$ (resolution of unity)

affine coherent state quantization: $$f \mapsto \hat{A}_f = \int_{\Pi_+} \frac{\mathrm{d}q\mathrm{d}p}{2\pi c_0(\xi_0)} |q,p\rangle f(q,p) \langle q,p|$$

$\longrightarrow \quad A_1 = \mathbb{1}$

$\longrightarrow \quad \forall \alpha \in \mathbb{R}, \quad \hat{A}_{q^\alpha} = \frac{c_\alpha(\xi_0)}{c_0(\xi_0)} \hat{x}^\alpha \Longrightarrow \hat{A}_q = \frac{c_1(\xi_0)}{c_0(\xi_0)} \hat{x}$

$\longrightarrow \quad \hat{A}_p = \frac{c_1(\xi_0)}{c_0(\xi_0)} \hat{p}$

impose $c_1(\xi_0) = c_0(\xi_0)$ to ensure $[\hat{A}_q, \hat{A}_p] = i$

$\longrightarrow \quad \hat{A}_{qp} = \frac{c_2(\psi_0)}{c_0(\psi_0)} \hat{d}$ dilation operator

$\longrightarrow \quad \hat{H} = \hat{A}_{p^2} = \frac{c_2(\psi_0)}{c_0(\psi_0)} \left\{ p^2 + \left[ \frac{K}{c_2(\psi_0)} - \frac{3}{2} \right] \frac{1}{\hat{x}^2} \right\}$ hamiltonian

$$K = \int_0^\infty \frac{\mathrm{d}y}{y^2} \xi_0'(y)^2$$

$> 0$

$$\hat{H}_\nu = \hat{p}^2 + \frac{\nu^2 - \frac{1}{4}}{\hat{x}^2}$$

# Coherent state semiclassical framework

new fiducial state $|\psi_0\rangle$

$$|\, q,p\,\rangle = \hat{V}_{q,p}|\,\psi_0\,\rangle$$

$$\forall\alpha\in\mathbb{R},\ \langle\, q,p\,|\hat{x}^{\alpha}|\,q,p\,\rangle = c_{-\alpha-2}(\psi_0)\, q^{\alpha}$$

$$\left.\begin{aligned}\langle\, q,p\,|\hat{x}|\,q,p\,\rangle &= c_{-3}(\psi_0)\, q\\ \langle\, q,p\,|\hat{p}|\,q,p\,\rangle &= c_{-3}(\psi_0)\, p\end{aligned}\right\}$$

Rescale $\psi_0;\, c_{-3}(\psi_0)\to 1$

(conformal) time-dependent functions $q(\eta)$ and $p(\eta)$

define time-dependent coherent states $|\, q(\eta),p(\eta)\,\rangle = \hat{V}_{q_\eta,p_\eta}|\,\psi_0\,\rangle$

$$\langle\hat{x}\rangle = q(\eta)$$

$$\langle\hat{p}\rangle = p(\eta)$$

→ $\hat{\mathcal{O}}(q,p)\mapsto \langle\, q(\eta),p(\eta)\,|\hat{\mathcal{O}}|\,q(\eta),p(\eta)\,\rangle = \mathcal{O}(\eta)$

→ semiclassical hamiltonian: $\langle\, q,p\,|\hat{p}^2|\,q,p\,\rangle = c_{-4}(\psi_0)\,p^2 + \dfrac{C}{q^2}$

$$C = \int_0^{\infty}\psi_0'(x)^2\,\mathrm{d}x > 0$$

# Choosing the fiducial state $|\psi_0\rangle$ wisely…

$$\hat{H}_\nu = \hat{p}^2 + \frac{\nu^2 - \frac{1}{4}}{\hat{x}^2}$$

1) demand that it solve the Schrödinger equation

$$i\partial_t \left[e^{-i\phi(\eta)} | q_\eta, p_\eta \rangle_{\psi_0}\right] = e^{-i\phi(\eta)} \hat{H}_\nu | q_\eta, p_\eta \rangle_{\psi_0}$$

with extra time dependent phase

1) apply Ehrenfest theorem:

$$\frac{\mathrm{d}}{\mathrm{d}\eta} \langle \psi(\eta) | \hat{\mathcal{O}} | \psi(\eta) \rangle = \mathrm{i} \langle \psi(\eta) | [\hat{H}_\nu, \hat{\mathcal{O}}] | \psi(\eta) \rangle$$

$\rightarrow e^{-i\phi(\eta)} | q_\eta, p_\eta \rangle_{\psi_0}$

effective e.o.m $\rightarrow$

$$\frac{\mathrm{d}}{\mathrm{d}\eta} \langle \psi(\eta) | \hat{x} | \psi(\eta) \rangle = \frac{\mathrm{d}q_\eta}{\mathrm{d}\eta} = 2 \langle \psi(\eta) | \hat{p} | \psi(\eta) \rangle = 2 p_\eta$$

$$\frac{\mathrm{d}}{\mathrm{d}\eta} \langle \psi(\eta) | \hat{p} | \psi(\eta) \rangle = \frac{\mathrm{d}p_\eta}{\mathrm{d}\eta} = 2 \left(\nu^2 - \frac{1}{4}\right) \langle \psi(\eta) | \hat{x}^{-3} | \psi(\eta) \rangle = 2 \left(\nu^2 - \frac{1}{4}\right) \frac{c_1(\psi_0)}{q_\eta^3}$$

$$\frac{\mathrm{d}}{\mathrm{d}\eta} \langle \psi(\eta) | \hat{H}_\nu | \psi(\eta) \rangle = \frac{\mathrm{d}}{\mathrm{d}\eta} \left[c_{-4}(\psi_0) p_\eta^2 + \frac{\left(\nu^2 - \frac{1}{4}\right) c_0(\psi_0) + C}{q_\eta^2}\right] = 0$$

# Choosing the fiducial state $|\psi_0\rangle$ wisely…

$$\hat{H}_\nu = \hat{p}^2 + \frac{\nu^2 - \frac{1}{4}}{\hat{x}^2}$$

semi classical hamiltonian: $$H_{\rm sc}(q_\eta, p_\eta) = p_\eta^2 + \frac{\left(\nu^2 - \frac{1}{4}\right) c_1(\psi_0)}{q_\eta^2}$$

$$\longrightarrow \left\{ \begin{aligned} \frac{\mathrm{d}q_\eta}{\mathrm{d}\eta} &= \frac{\partial H_{\rm sc}}{\partial p_\eta} \\ \frac{\mathrm{d}p_\eta}{\mathrm{d}\eta} &= -\frac{\partial H_{\rm sc}}{\partial q_\eta} \end{aligned} \right.$$

**+ constraint:** $$C = \int_0^\infty \psi_0'(x)^2 \mathrm{d}x = \left(\nu^2 - \frac{1}{4}\right)\left[c_1(\psi_0)c_{-4}(\psi_0) - c_0(\psi_0)\right]$$

***POSSIBLE ???***

# Choosing the fiducial state $|\psi_0\rangle$ wisely…

$$\hat{H}_\nu = \hat{p}^2 + \frac{\nu^2 - \frac{1}{4}}{\hat{x}^2}$$

3) Necessary condition: $$i\,\hat{V}^\dagger_{q_\eta p_\eta}\partial_\eta\left[e^{-i\phi(\eta)}\hat{V}_{q_\eta p_\eta}\right]|\,\psi_0\,\rangle = e^{-i\phi(\eta)}\hat{V}^\dagger_{q_\eta p_\eta}\hat{H}_\nu\hat{V}_{q_\eta p_\eta}|\,\psi_0\,\rangle$$

$$e^{i\phi(\eta)}i\,\hat{V}^\dagger_{q_\eta p_\eta}\partial_\eta\left[e^{-i\phi(\eta)}\hat{V}_{q_\eta p_\eta}\right] = \phi'(\eta) + p_\eta^2\hat{x}^2 - \frac{\left(\nu^2 - \frac{1}{4}\right)c_1(\psi_0)}{q_\eta^2}\,\hat{x}^2 + \frac{2p_\eta}{q_\eta}\hat{d}$$

$$\hat{V}^\dagger_{q_t p_t}\hat{H}_\nu\hat{V}_{q_t p_t} = \frac{1}{q_t^2}\hat{p}^2 + p_t^2\hat{x}^2 + \frac{2p_t}{q_t}\hat{d} + \frac{\nu^2 - \frac{1}{4}}{q_t^2\,\hat{x}^2}$$

$$\Longrightarrow \quad \left[\hat{p}^2 + \frac{\nu^2 - \frac{1}{4}}{\hat{x}^2} + \left(\nu^2 - \frac{1}{4}\right)c_1(\psi_0)\,\hat{x}^2\right]|\,\psi_0\,\rangle = q_\eta^2\phi'(\eta)|\,\psi_0\,\rangle$$

eigenvalue $\omega$

# Choosing the fiducial state $|\psi_0\rangle$ wisely…

$$\hat{H}_\nu = \hat{p}^2 + \frac{\nu^2 - \frac{1}{4}}{\hat{x}^2}$$

4) Solve eigenvalue equation

$$\left[\hat{p}^2 + \frac{\nu^2 - \frac{1}{4}}{\hat{x}^2} + \left(\nu^2 - \frac{1}{4}\right) c_1(\psi_0)\,\hat{x}^2\right] |\,\psi_0\,\rangle = q_t^2 \phi'(\eta) |\,\psi_0\,\rangle$$

$$e^{-i\phi(\eta)} = \left[\frac{\sqrt{\left(\nu^2 - \frac{1}{4}\right) c_1(\psi_0)} - i q_\eta p_\eta}{\sqrt{\left(\nu^2 - \frac{1}{4}\right) c_1(\psi_0)} + i q_\eta p_\eta}\right]^{\frac{1}{4}\omega\left[\left(\nu^2 - \frac{1}{4}\right) c_1(\psi_0)\right]^{-1/2}}$$

using $\dfrac{\mathrm{d}}{\mathrm{d}\eta}(q_\eta p_\eta) = 2H_{\mathrm{sc}}(q_\eta, p_\eta)$

5) Solve eigenfunction equation

$$\hat{H}_0 |\,\Phi_n\,\rangle = \left(\hat{p}^2 + \frac{\nu^2 - \frac{1}{4}}{\hat{x}^2} + \xi_\nu^2 \hat{x}^2\right) |\,\Phi_n\,\rangle = \omega_n |\,\Phi_n\,\rangle$$

$\nu > \dfrac{1}{2}$ $\qquad$ $\xi_\nu > 0$

$\longrightarrow \quad \Phi_n(x) = \langle\, x \,|\, \Phi_n \,\rangle \in \mathbb{R}$

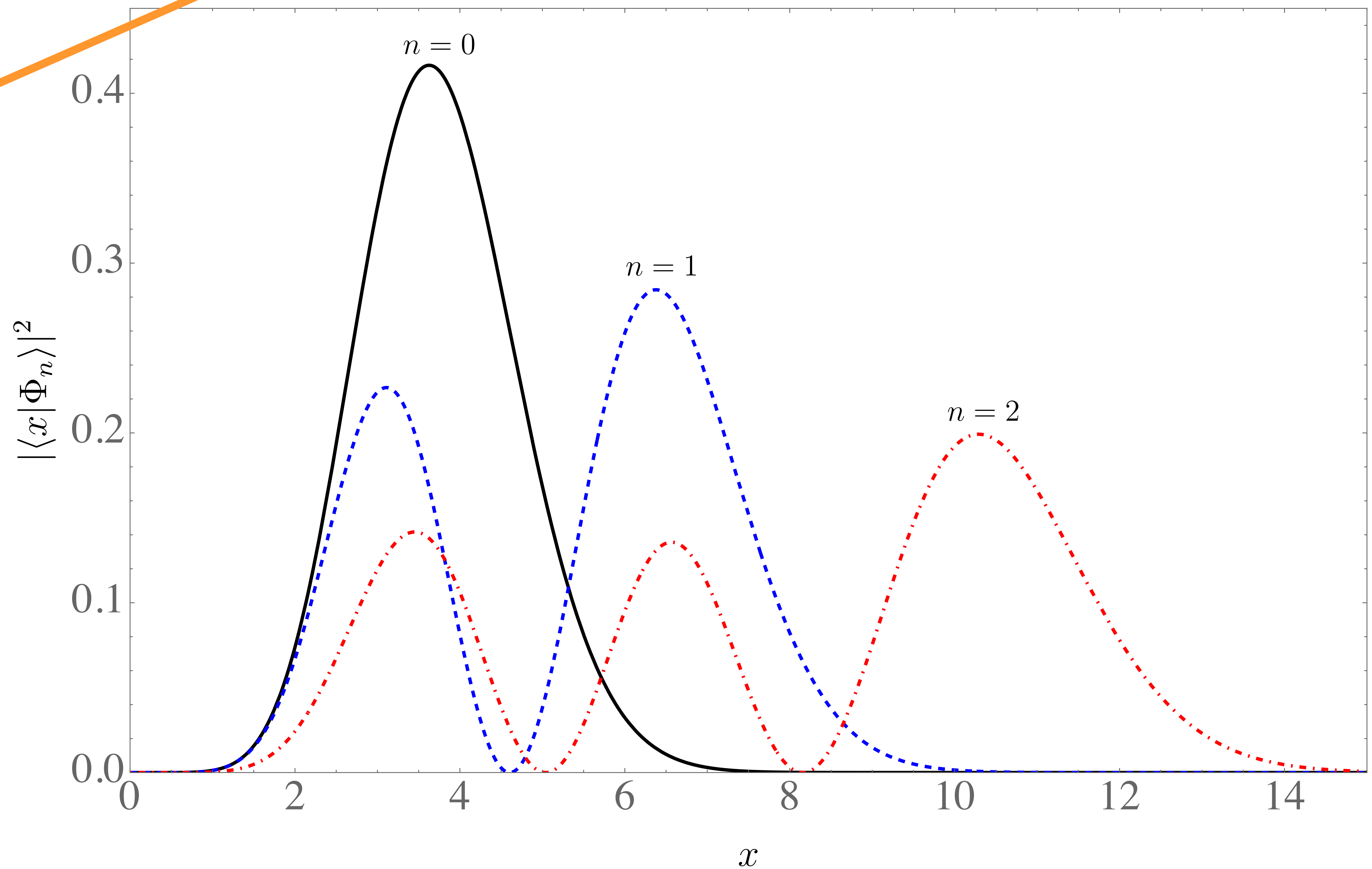
# Choosing the fiducial state $|\psi_0\rangle$ wisely…

6) Open book on known hamiltonian eigenvalues and eigenvectors:

$$\longrightarrow \quad \omega_n = 2\xi_\nu(2n+\nu+1)$$

$$\longrightarrow \quad \Phi_n(x) = \sqrt{\frac{2\ n!}{\Gamma(\nu+n+1)}}\ \xi_\nu^{\frac{\nu+1}{2}}\ x^{\nu+1/2} L_n^\nu\left(\xi_\nu x^2\right) e^{-\frac{1}{2}\xi_\nu x^2}$$

Laguerre polynomials



# Choosing the fiducial state $|\psi_0\rangle$ wisely…

7) Wrap up with full solutions

$$|\, q, p; \nu, n\,\rangle \stackrel{\text{def}}{=} e^{-i\phi_{q,p}} |\, q, p\,\rangle_{\psi_0}$$

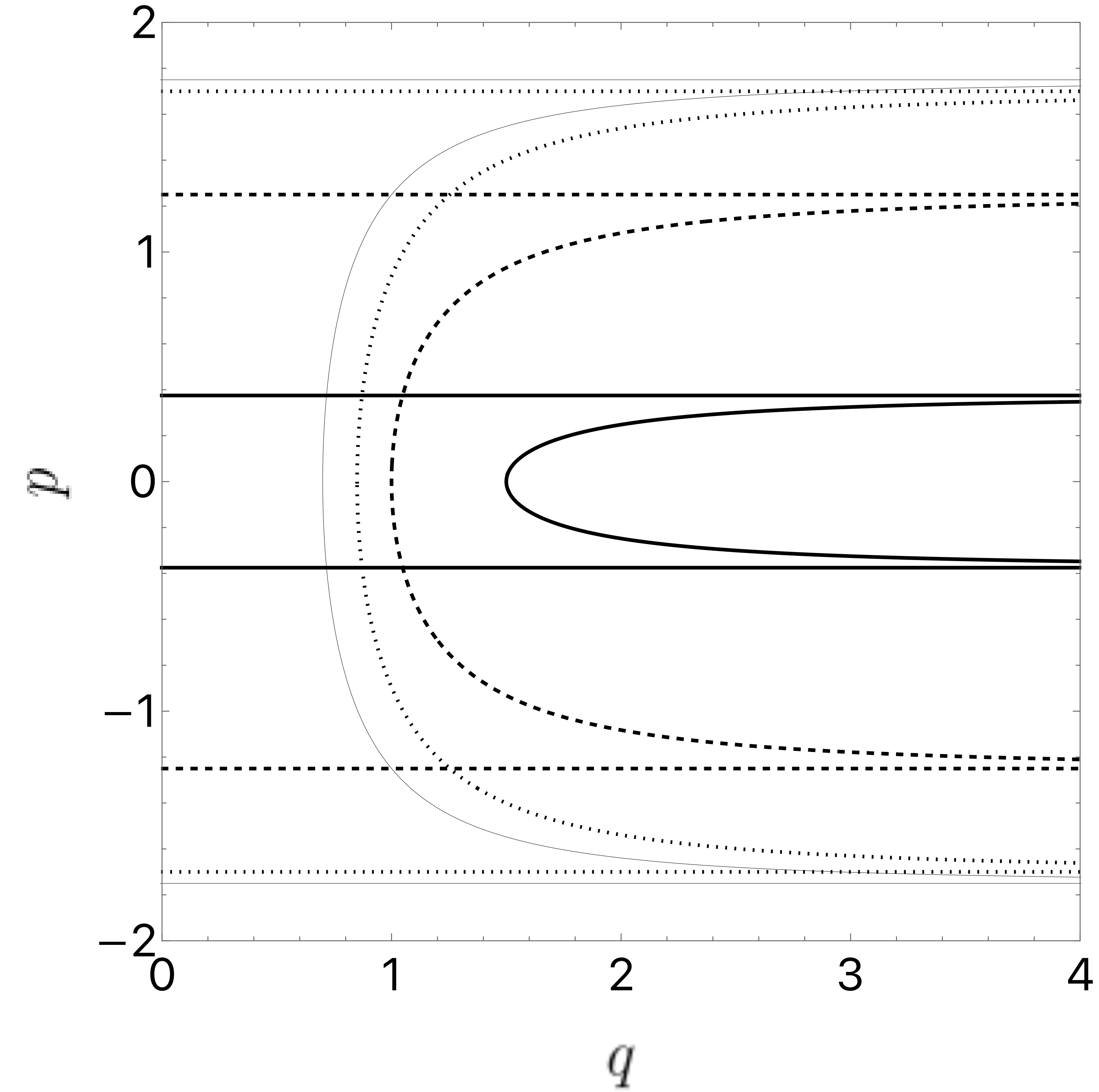
$$\longrightarrow \quad e^{-i\phi(\eta)} = \left(\frac{\xi_\nu - i q_\eta p_\eta}{\xi_\nu + i q_\eta p_\eta}\right)^{\frac{\omega}{4\xi_\nu}}$$

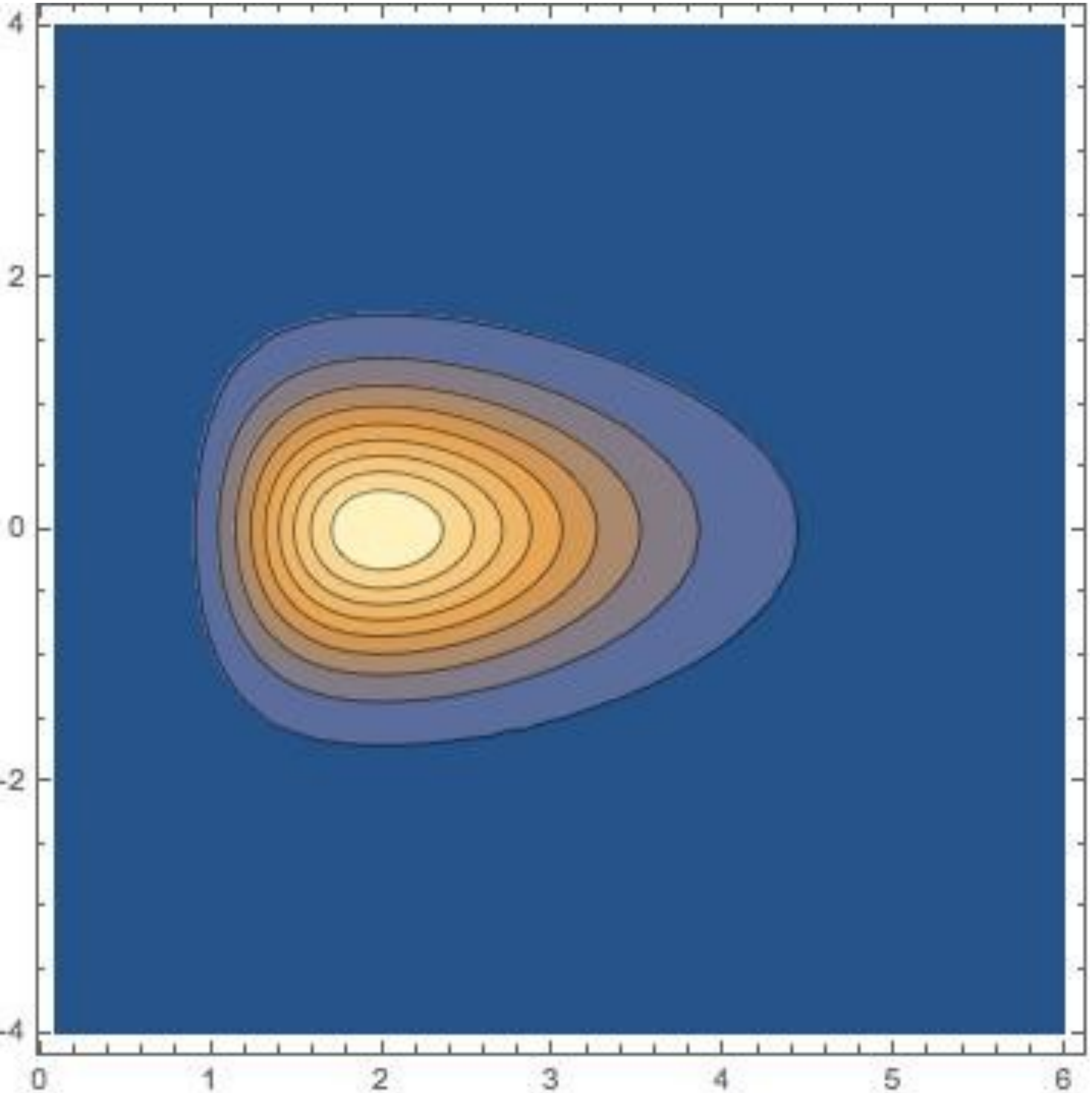
$$\langle\, x\,|\, q_\eta, p_\eta; \nu, n\,\rangle = \sqrt{\frac{2\; n!}{\Gamma(\nu+n+1)}\left(\frac{\xi_{\nu,n} - \mathrm{i} q_\eta p_\eta}{\xi_{\nu,n} + \mathrm{i} q_\eta p_\eta}\right)^{\frac{1}{2}(2n+\nu+1)}}\; \xi_{\nu,n}^{\frac{\nu+1}{2}} \frac{x^{\nu+1/2}}{q_\eta^{\nu+1}} L_n^\nu\left(\xi_{\nu,n}\frac{x^2}{q_\eta^2}\right) \exp\left[-\frac{1}{2}(\xi_{\nu,n} - \mathrm{i}\, q_\eta p_\eta)\frac{x^2}{q_\eta^2}\right]$$

8) Plot nice figures for the phase space evolution…

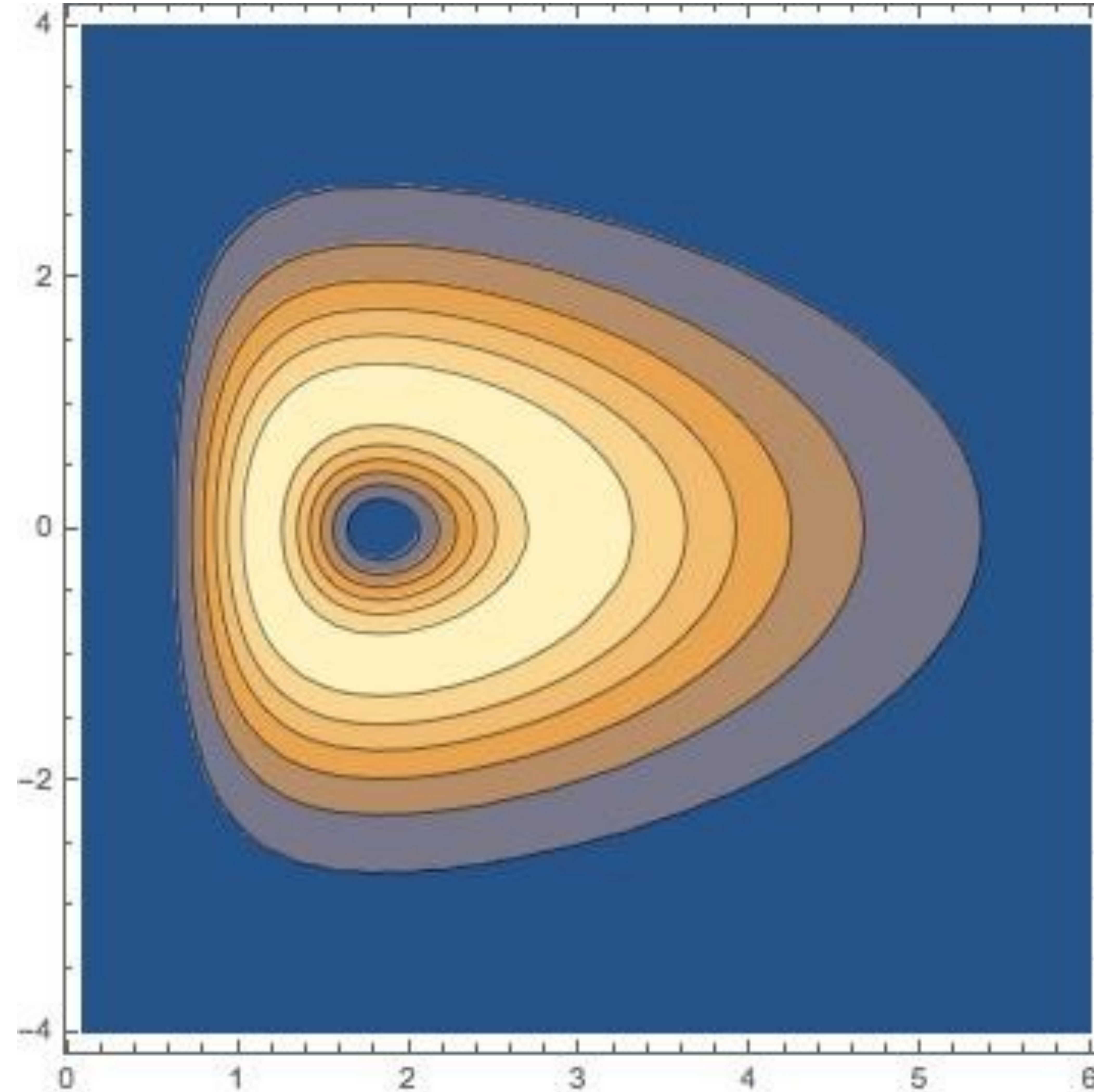
semiclassical trajectory in phase space

$$\begin{cases} q(\eta) = q_{\mathrm{B}}\sqrt{1 + \left(\dfrac{\eta - \eta_{\mathrm{B}}}{\bar\eta}\right)^2} \\ p(\eta) = \dfrac{1}{2}\dfrac{\mathrm{d}q}{\mathrm{d}\eta} = \dfrac{q_{\mathrm{B}}^2}{2\bar\eta q(\eta)}\left(\dfrac{\eta - \eta_{\mathrm{B}}}{\bar\eta}\right) \end{cases}$$



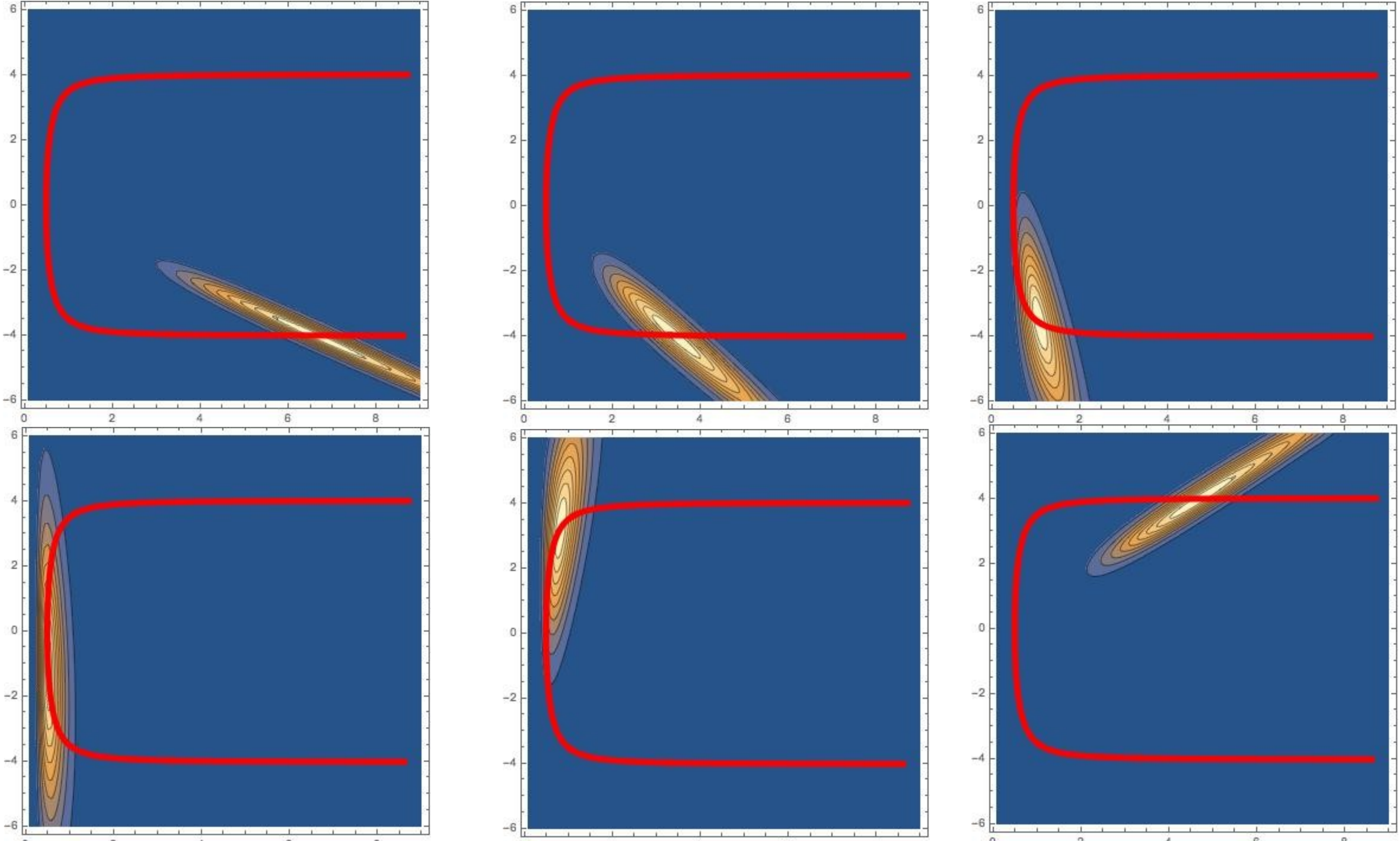



$q = 2, p = 0, \nu = 3, n = 0$




$q = 2, p = 0, \nu = 3, n = 1$

$\rho_\psi(q,p) = |\langle\, q, p; \nu, 0 \,|\, \psi \,\rangle|^2 / [2\pi c_0(\nu, 0)]$ semi classical probability density



6
4
2
0
−2
−4
−6
0
2
4
6
8

Next step: include perturbations $$H = H^{(0)} + \sum_{\boldsymbol{k}} H_{\boldsymbol{k}}^{(2)} = H^{(0)} + \sum_{\boldsymbol{k}} \frac{1}{2}|\pi_{v,\boldsymbol{k}}|^2 + \frac{1}{2}\left(wk^2 - \mathcal{V}\right)|v_{\boldsymbol{k}}|^2$$

$$\mathcal{V} = \frac{8(1-3w)}{(1+3w)^2}\frac{p^2}{q^2}$$

Mukhanov variable

$$\Longrightarrow \quad i\frac{\partial}{\partial\eta}|\Psi\rangle = \left(\widehat{H}^{(0)} + \widehat{H}^{(2)}\right)|\Psi\rangle$$

<u>usual way to go</u> Born-Oppenheimer expansion $|\Psi^{(\mathrm{BO})}\rangle = |\psi^{(\mathrm{bg})}(q)\rangle \otimes |\psi^{(\mathrm{pert})}\left[\{v_{\boldsymbol{k}}\};q(\eta),p(\eta)\right]\rangle$

+ Schrödinger $$i\frac{\partial}{\partial\eta}|\psi^{(\mathrm{pert})}\rangle = \langle\psi^{(\mathrm{bg})}|\widehat{H}^{(2)}|\psi^{(\mathrm{bg})}\rangle\,|\psi^{(\mathrm{pert})}\rangle$$

but possible non diagonal terms… $\langle\psi_1^{(\mathrm{bg})}|\widehat{H}^{(2)}|\psi_2^{(\mathrm{bg})}\rangle$

BO not stable $|\Psi\rangle = |\psi_0^{(\mathrm{bg})}\rangle|\psi_0^{(\mathrm{pert})}\rangle$ —time evolution→ $|\Psi(\eta)\rangle = \sum_n |\psi_n^{(\mathrm{bg})}(\eta)\rangle|\psi_n^{(\mathrm{pert})}(\eta)\rangle$

BO initial condition — interferences

Schrödinger $\Longrightarrow$ $i\frac{\partial}{\partial\eta}|\psi_n^{(\mathrm{pert})}\rangle = \sum_{\ell m} M_{n\ell}^{-1}\widehat{H}_{\ell m}^{(2)}|\psi_m^{(\mathrm{pert})}\rangle$

$M_{nm} = \langle\psi_n^{(\mathrm{bg})}|\psi_m^{(\mathrm{bg})}\rangle$ — $\langle\psi_\ell^{(\mathrm{bg})}|\widehat{H}^{(2)}|\psi_m^{(\mathrm{bg})}\rangle$

$\rho_{\tilde{q},\tilde{p}}(q,p) = |\langle q,p|\tilde{q},\tilde{p}\rangle|^2$ overcomplete basis $\langle\psi_n^{(\mathrm{bg})}|\widehat{H}^{(2)}|\psi_0^{(\mathrm{bg})}\rangle \neq 0$

no “δ”

***a simple `semiclassical' state eventually evolves into a multibranched state***

interferences / virtual universes / loop expansion

interpretation / meaning?

BO fixed basis $\{|\phi_m\rangle\}$ such that $i\partial_\eta|\phi_n\rangle = \langle\psi_n^{(\mathrm{bg})}|\widehat{H}^{(2)}|\psi_n^{(\mathrm{bg})}\rangle|\phi_n\rangle$ (same $n$)

wave function decomposition $|\psi_n^{(\mathrm{pert})}\rangle = \sum_m \alpha_{nm}|\phi_m\rangle$

$\Longrightarrow$ time evolution

$$i\alpha'_{nm} = \sum_{\ell rkp} M^{-1}_{n\ell} S^{-1}_{mk} H^{(2)}_{\ell rkp}\alpha_{rp} - \sum_{kp} S^{-1}_{mk} H^{(2)}_{ppkp}\alpha_{np}$$

$$S_{kp} = \langle\phi_k|\phi_p\rangle \qquad \langle\phi_k|\widehat{H}^{(2)}_{\ell m}|\phi_p\rangle$$

multiverse amplitude evolution

Universe → **biverse** → multiverse

fixed basis

$$\left.\begin{aligned}|\psi_0^{(\mathrm{bg})}(\eta)\rangle &= |q_0(\eta), p_0(\eta)\rangle \\ |\psi_1^{(\mathrm{bg})}(\eta)\rangle &= |q_1(\eta), p_1(\eta)\rangle\end{aligned}\right\}$$ two background modes with energies $E_0$ and $E_1$

Born-Oppenheimer basis $|\phi_{E_0}(\eta)\rangle$ and $|\phi_{E_1}(\eta)\rangle$ →

$$\begin{cases} i\partial_\eta|\phi_{E_0}\rangle = \langle q_0(\eta), p_0(\eta)|\widehat{H}_{\boldsymbol{k}}^{(2)}|q_0(\eta), p_0(\eta)\rangle|\phi_{E_0}\rangle \\ i\partial_\eta|\phi_{E_1}\rangle = \langle q_1(\eta), p_1(\eta)|\widehat{H}_{\boldsymbol{k}}^{(2)}|q_1(\eta), p_1(\eta)\rangle|\phi_{E_1}\rangle \end{cases}$$

perturbation basis

$$|\psi_0^{(\mathrm{pert})}(\eta)\rangle = \alpha_{00}(\eta)|\phi_{E_0}(\eta)\rangle + \alpha_{01}(\eta)|\phi_{E_1}(\eta)\rangle$$

$$|\psi_1^{(\mathrm{pert})}(\eta)\rangle = \alpha_{10}(\eta)|\phi_{E_0}(\eta)\rangle + \alpha_{11}(\eta)|\phi_{E_1}(\eta)\rangle$$

$$\Longrightarrow |\Psi(\eta)\rangle = |q_0, p_0\rangle\left[\alpha_{00}(\eta)|\phi_{E_0}(\eta)\rangle + \alpha_{01}(\eta)|\phi_{E_1}(\eta)\rangle\right] + |q_1, p_1\rangle\left[\alpha_{10}(\eta)|\phi_{E_0}(\eta)\rangle + \alpha_{11}(\eta)|\phi_{E_1}(\eta)\rangle\right]$$

One $k$ mode perturbation hamiltonian $$\hat{H}_{ij}^{(2)} = \left(-\frac{1}{2}\frac{\partial^2}{\partial v_{\boldsymbol{k}}^2} + \frac{1}{2} w k^2 v_{\boldsymbol{k}}^2\right) M_{ij} - \frac{1}{2}\mathcal{V}_{ij} v_{\boldsymbol{k}}^2$$

$$\langle q_i, p_i | q_j, p_j \rangle = \int \mathrm{d}x \langle q_i, p_i | x \rangle \langle x | q_j, p_j \rangle$$

$$\langle q_i, p_i | \widehat{\mathcal{V}} | q_j, p_j \rangle$$

Analytical integrations

$$M_{ij} = \left[\frac{2\left(\frac{\xi_\nu + i p_i q_i}{\xi_\nu - i p_i q_i}\right)^{1/2}\left(\frac{\xi_\nu - i p_j q_j}{\xi_\nu + i p_j q_j}\right)^{1/2}}{\left(\frac{q_j}{q_i} + \frac{q_i}{q_j}\right) + \frac{i}{\xi_\nu}\left(p_i q_j - p_j q_i\right)}\right]^{\nu+1}$$

$$\mathcal{V}_{ij} = \frac{2(1-3w)}{(1+3w)^2 \nu(\nu-1)}\left\{(\nu+1)(\nu-3)\left[\frac{p_i^2}{q_i^2} + \frac{p_j^2}{q_j^2} + 2i\xi_\nu\left(\frac{p_i}{q_i^3} - \frac{p_j}{q_j^3}\right) - \xi_\nu^2\left(\frac{1}{q_i^4} + \frac{1}{q_j^4}\right)\right] + (\nu^2+3)\left[2\frac{p_i}{q_i}\frac{p_j}{q_j} - 2i\xi_\nu\left(\frac{p_i}{q_i q_j^2} - \frac{p_j}{q_j q_i^2}\right) + \frac{\xi_\nu^2}{q_i^2 q_i^2}\right]\right\} M_{ij}$$

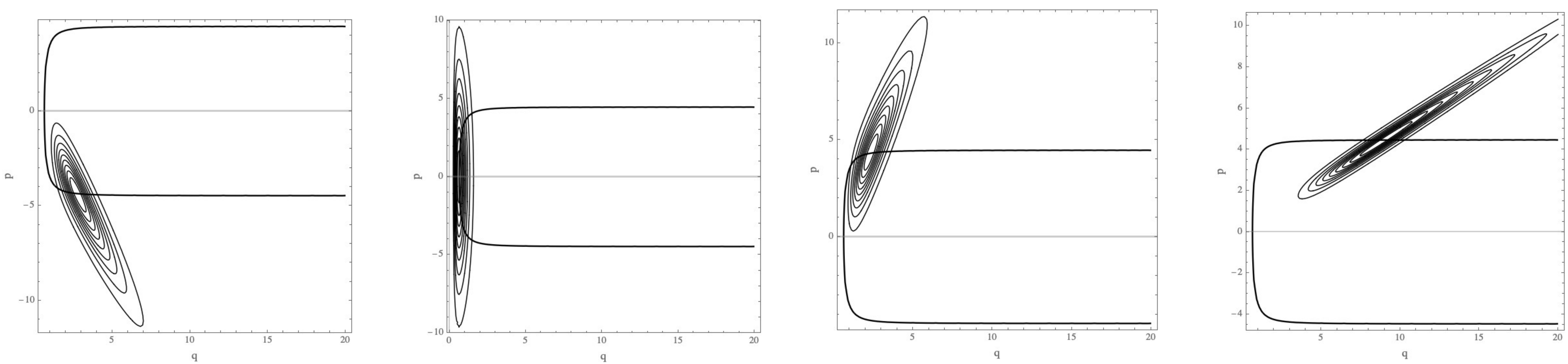
$$\Longrightarrow \quad \mathcal{V}_{ii} = \frac{8(1-3w)}{(1+3w)^2}\left[\frac{p_i^2}{q_i^2} + \frac{\nu+3}{\nu(\nu-1)}\frac{\xi_\nu^2}{q_i^4}\right]$$ diagonal element

Gaussian perturbation representation $\langle v_{\boldsymbol{k}} | \phi_{E_i} \rangle = \left[ \frac{2\Re\mathrm{e}(\Omega_i)}{\pi} \right]^{1/4} \exp\left[ -\Omega_i(\eta) v_{\boldsymbol{k}}^2 \right]$

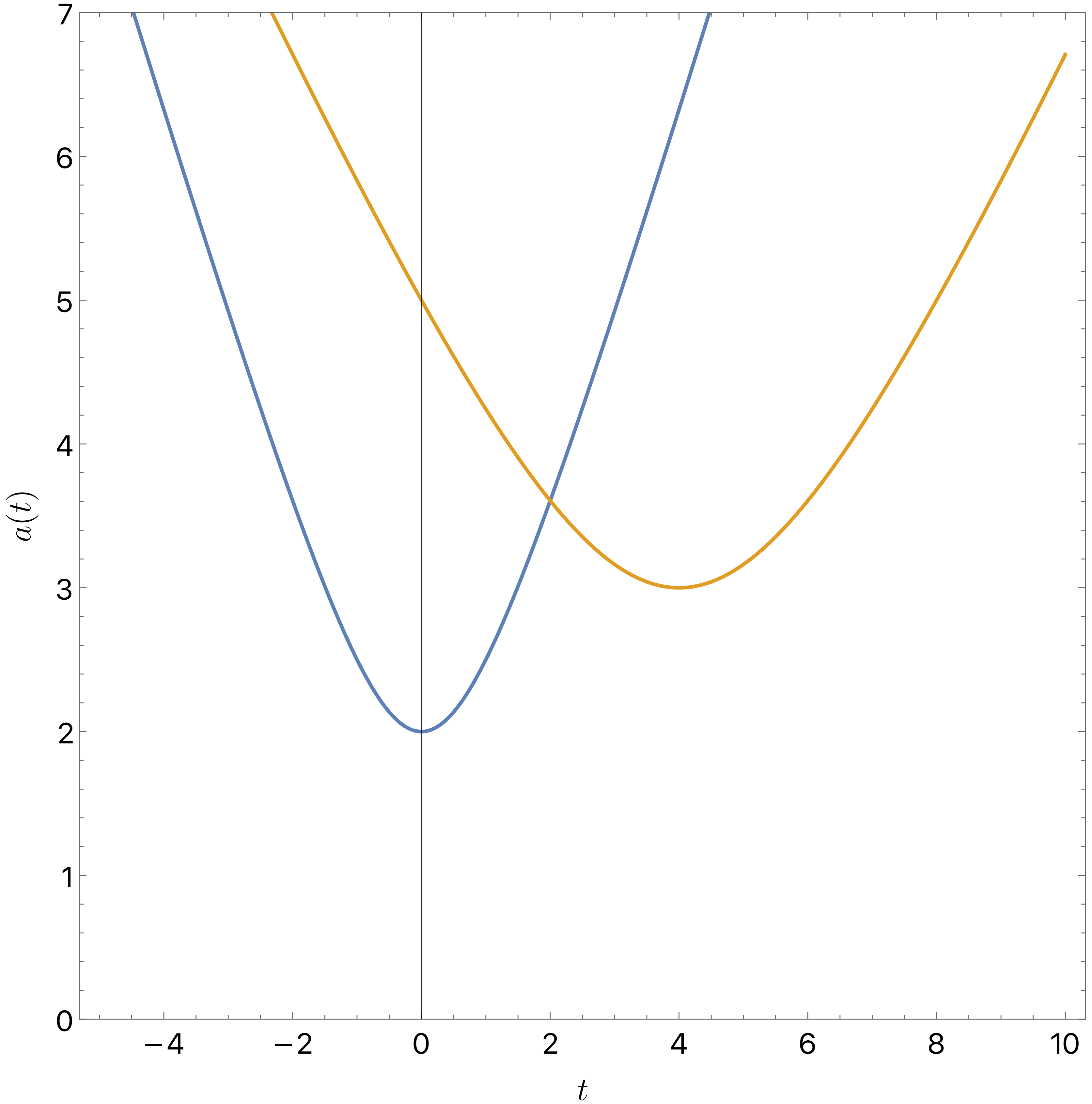
matrix elements $H_{ijkl}^{(2)} = \sqrt{2} M_{ij} \left[ \Re\mathrm{e}\left(\Omega_k\right) \Re\mathrm{e}\left(\Omega_l\right) \right]^{\frac{1}{4}} \left[ \frac{\Omega_l}{(\Omega_k^\star + \Omega_l)^{\frac{1}{2}}} - \frac{\Omega_l^2}{(\Omega_k^\star + \Omega_l)^{\frac{3}{2}}} + \frac{wk^2 - \mathcal{V}_{ij}/M_{ij}}{4(\Omega_k^\star + \Omega_l)^{\frac{3}{2}}} \right]$

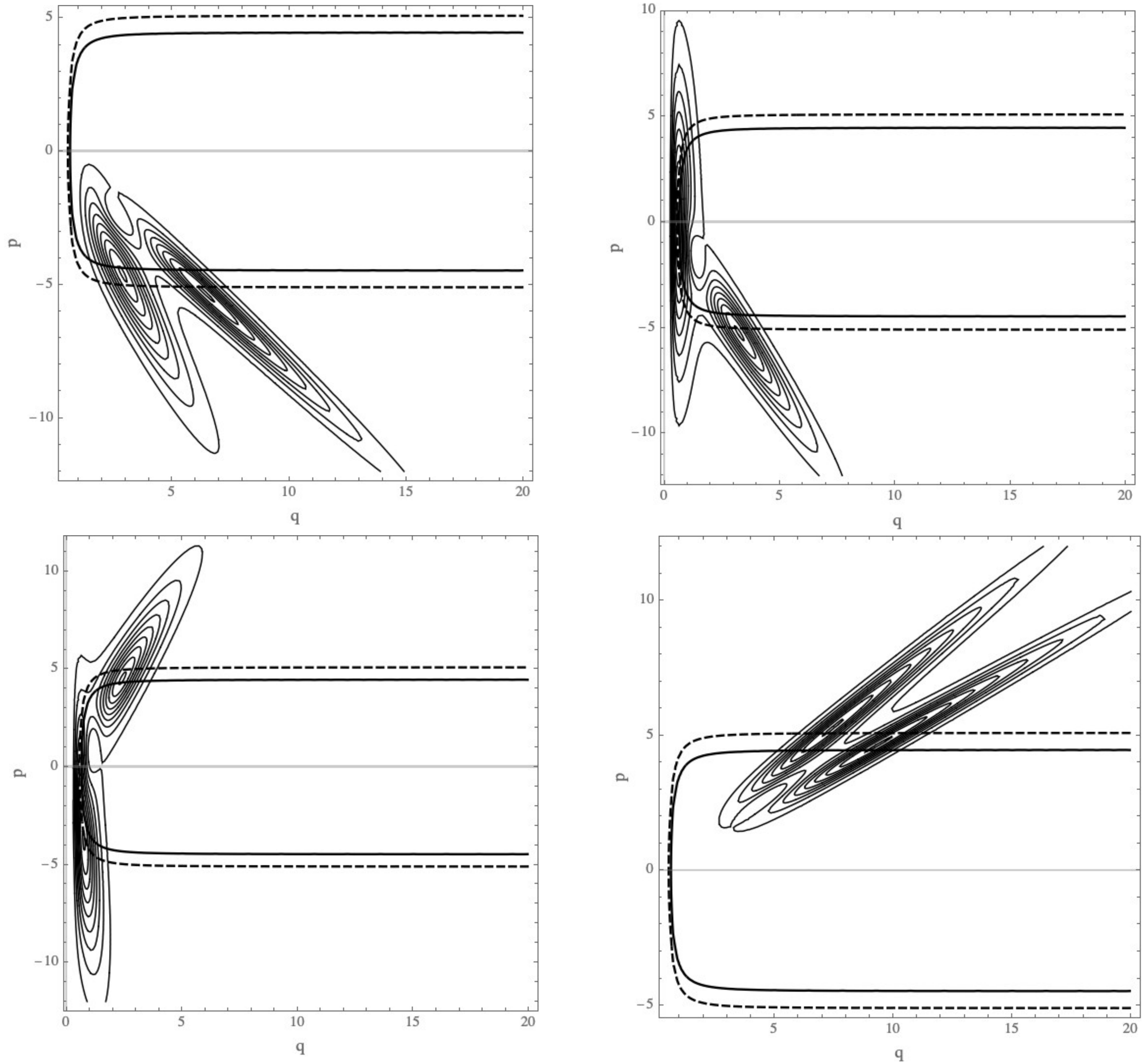
mode equation $f_i'' + (wk^2 - \mathcal{V}_{ii}) f_i = 0$ $\Longrightarrow$ variance $\Omega_i = -\frac{i}{2} \frac{f_i'}{f_i}$

$$\rho_E(q,p,\eta) = |\psi_E(q,p,\eta)|^2 = \left| \int_0^\infty \mathrm{d}x \langle q,p|x\rangle\langle x|\psi_E\rangle \right|^2$$

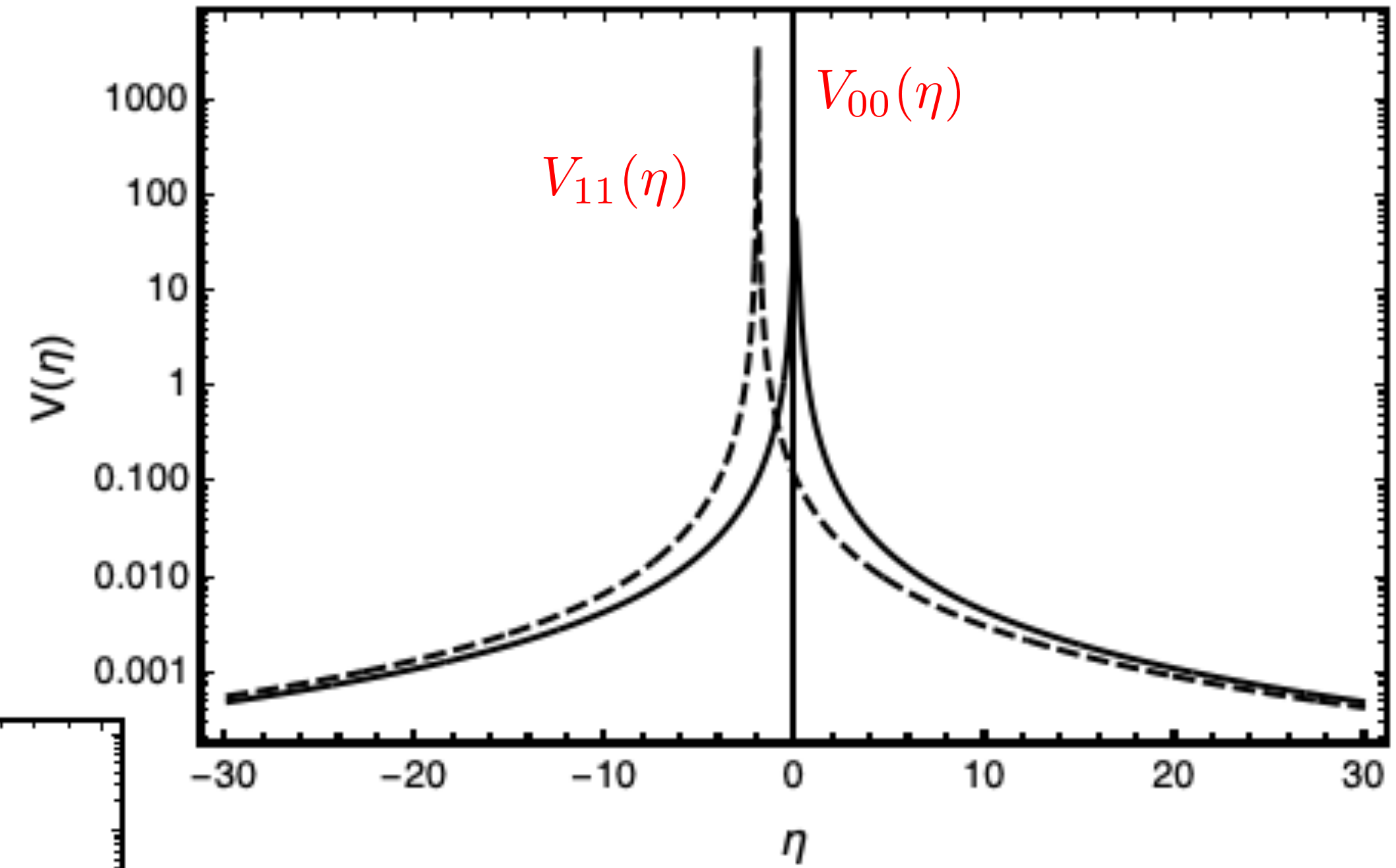


two modes $$\rho_{E_0,E_1}(q,p,\eta) = \frac{|\psi_{E_0}(q,p,\eta-\eta_0) + \psi_{E_1}(q,p,\eta-\eta_1)|^2}{2(1+\Re\mathrm{e}\langle q_0,p_0|q_1,p_1\rangle)}$$

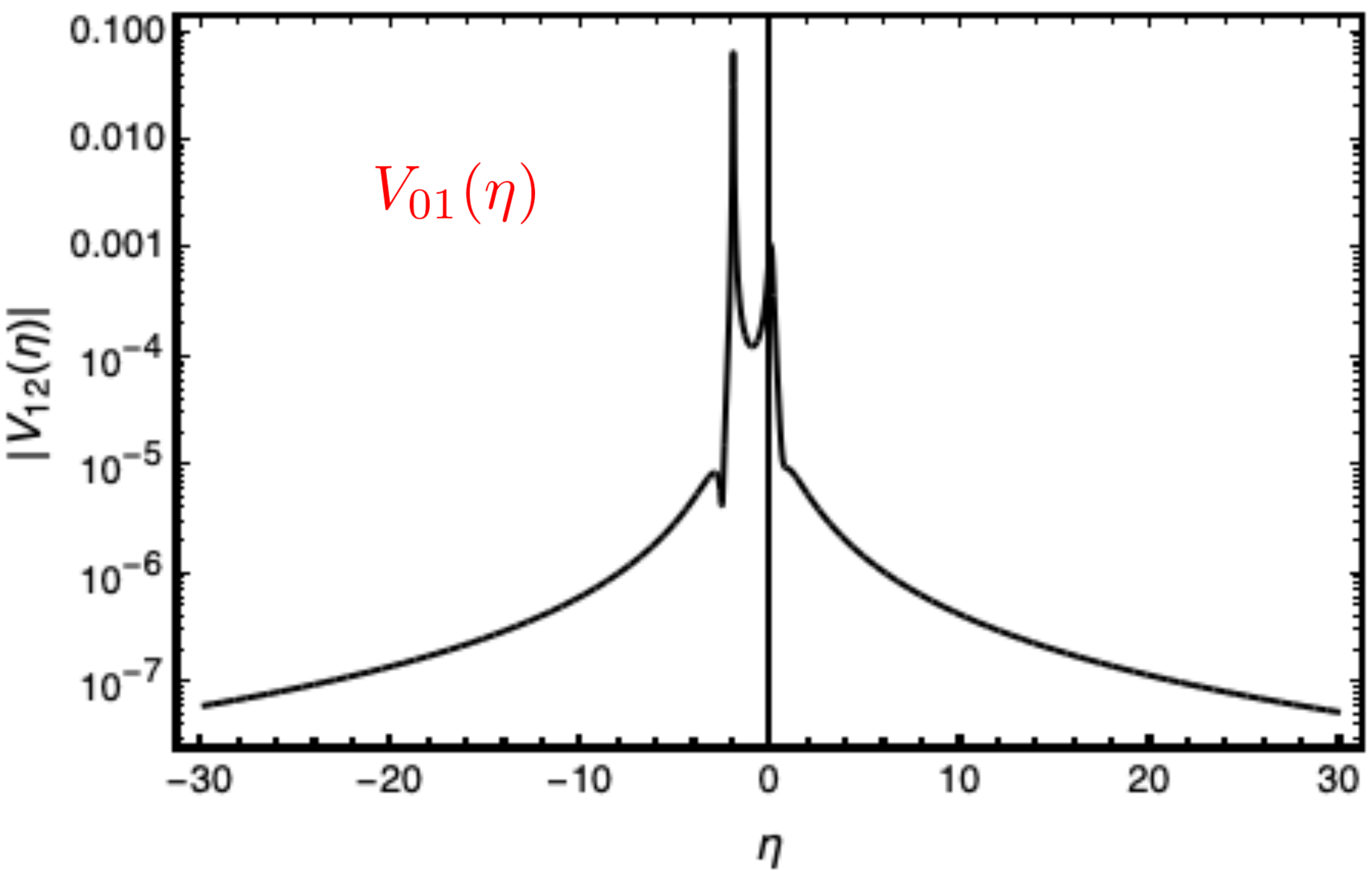


potentials

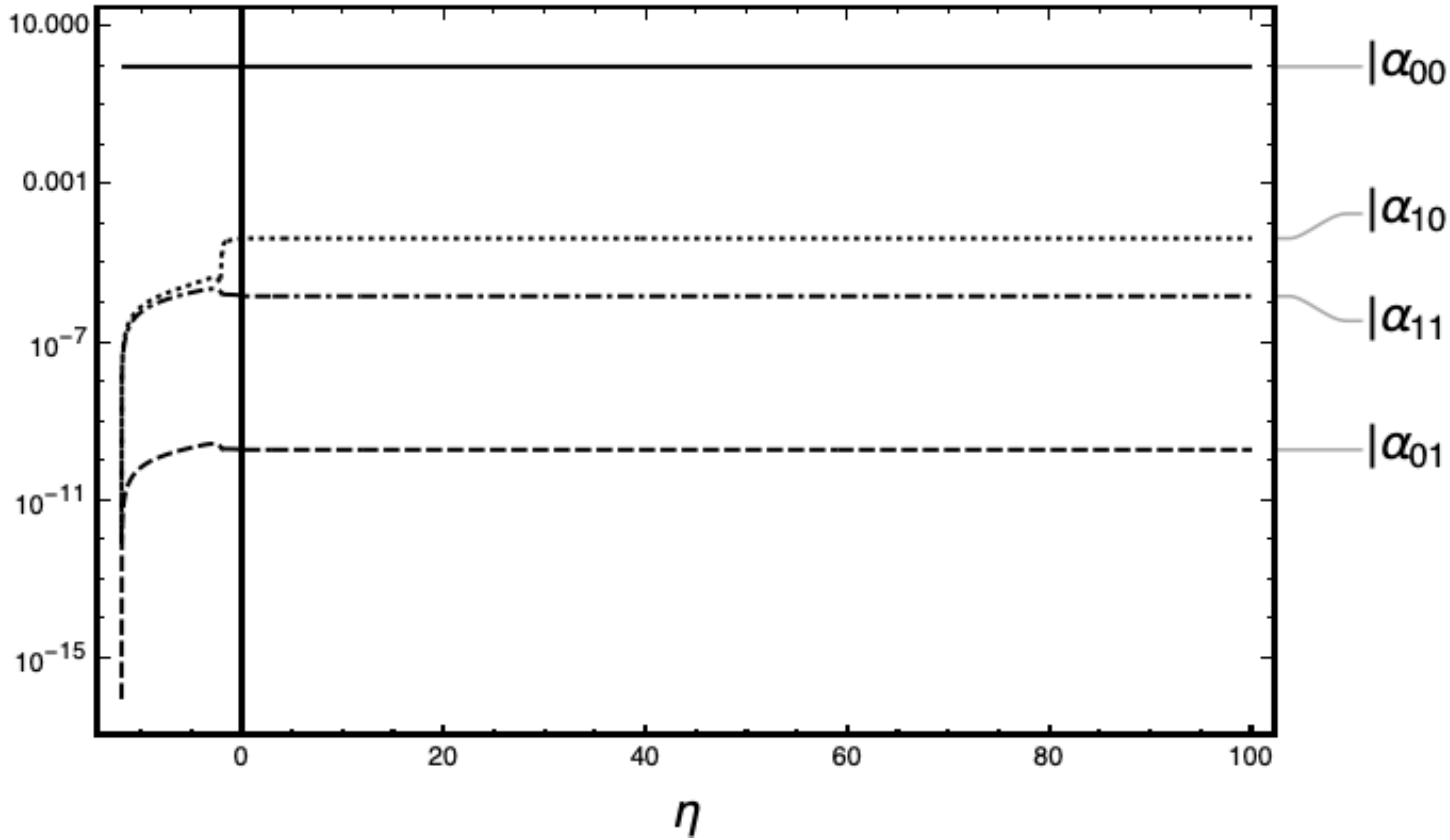


$V_{00}(\eta)$
$V_{11}(\eta)$
1000
100
10
1
0.100
0.010
0.001
$V(\eta)$
−30
−20
−10
0
10
20
30
$\eta$

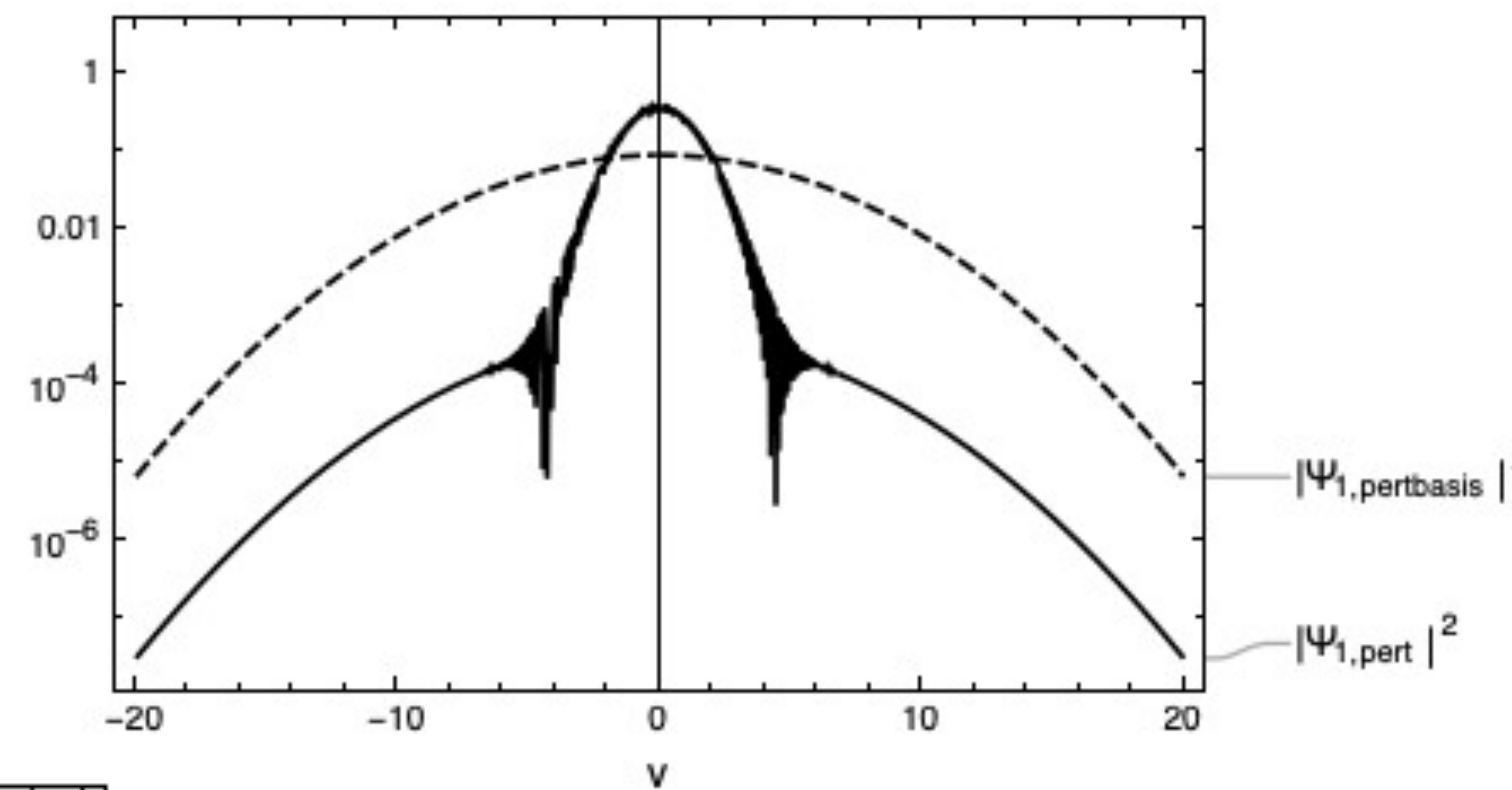



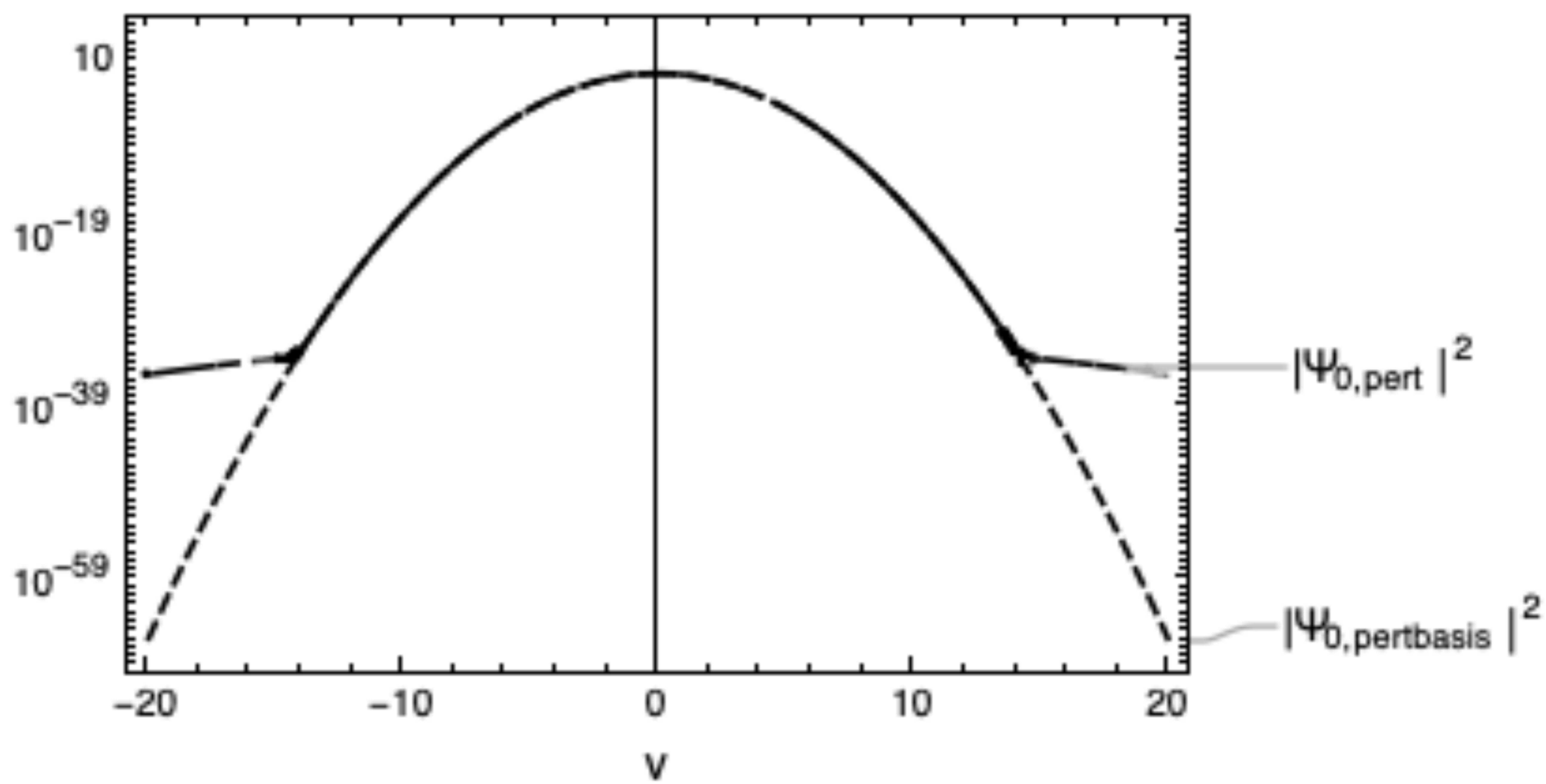
$V_{01}(\eta)$
0.100
0.010
0.001
$10^{-4}$
$10^{-5}$
$10^{-6}$
$10^{-7}$
$|V_{12}(\eta)|$
−30
−20
−10
0
10
20
30
$\eta$

time development of non-BO parameters



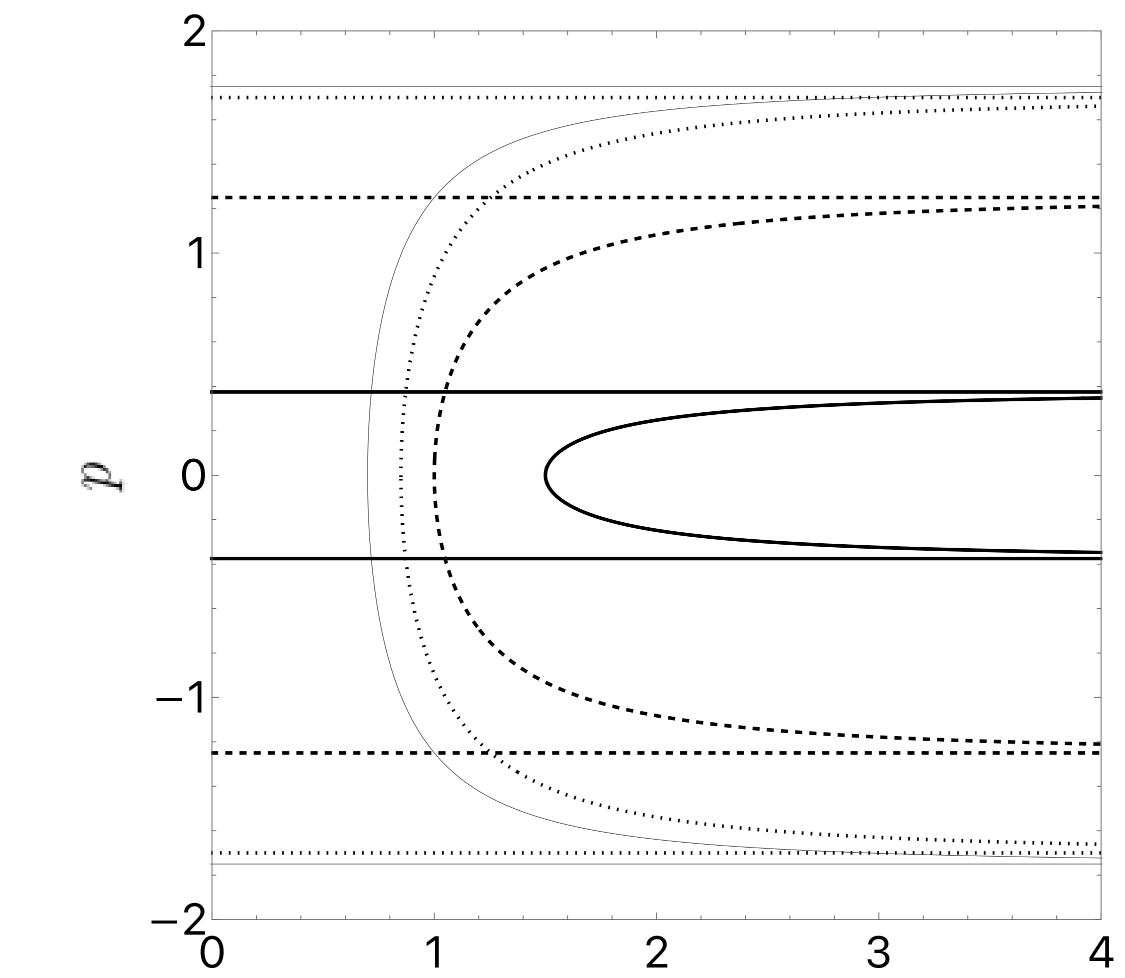
projected distribution



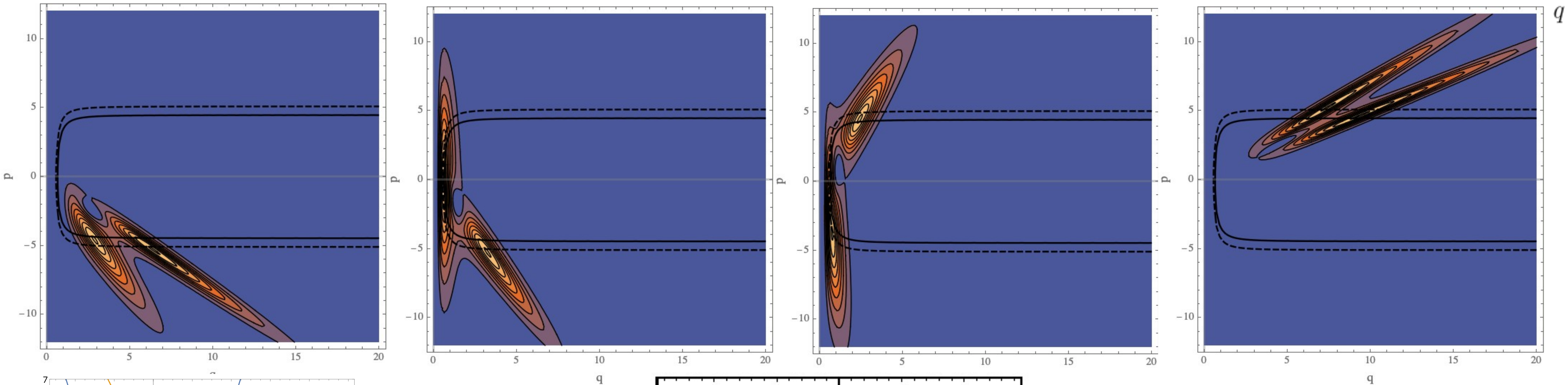

Non gaussian

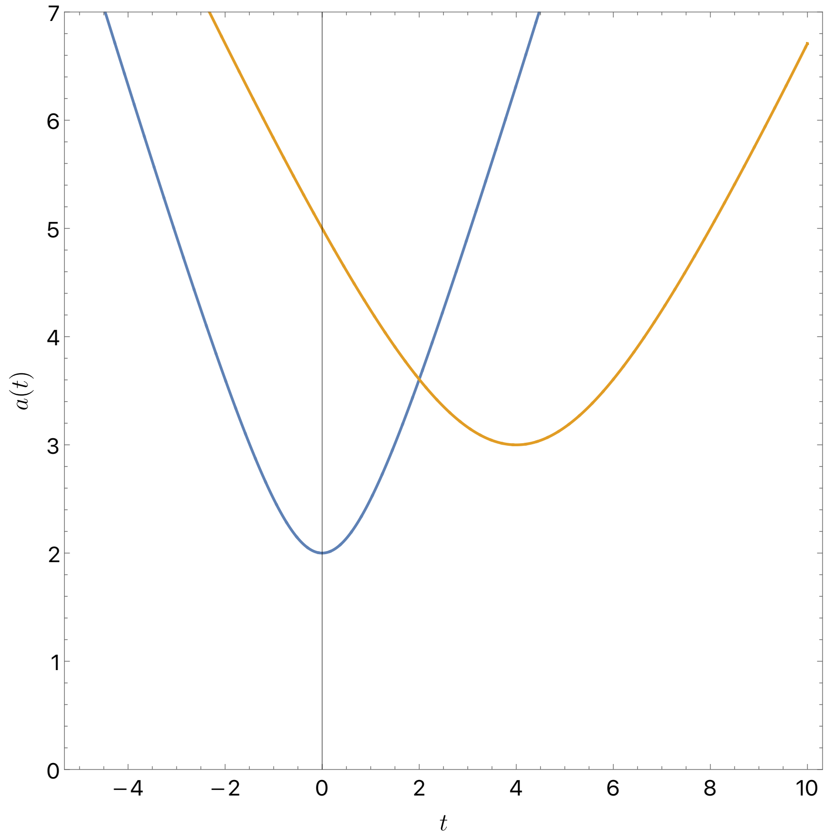
# Conclusions

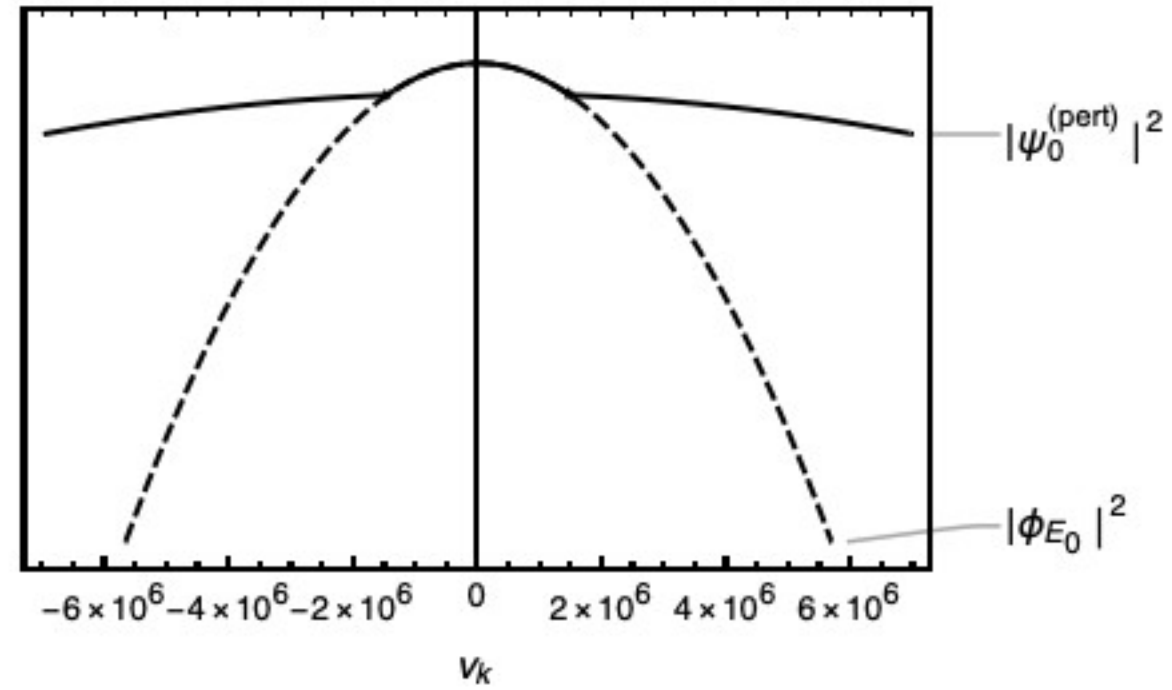



- **A new class of coherent state basis for background quantum cosmology**
- ***quantum* background + *quantum* fluctuations**

superposition principe → possible interferences




Non gaussianities
from gaussian states