M-theory, gauged supergravity and holography

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L2C Montpellier

Based on

[arXiv:2210.09318]

and [2407.13177]; [2312.08909]; [2304.01734]; [2106.04581]

with

N. Bobev & J. Hong,

A. Charles, S. Choi, K. Hristov

Introduction

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$$\log Z = \sum_{g\geq 0} N^{2-2g}\mathcal{F}_g(\lambda)$$

- The AdS/CFT correspondence relates observables

$$Z(\text{CFT}_{d-1}) = Z(\text{string/M on AdS}_d \times X_{D-d})$$

- Use CFT to learn about quantum gravity.

- Can we compute path integrals in CFTs?

$$Z(\mathrm{CFT}) = \langle \mathbb{1} \rangle = \int \mathcal{D}\Phi \, e^{-S[\Phi]} = \ldots ?$$

- "Keep calm and use SUSY".

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- "Keep calm and use SUSY".

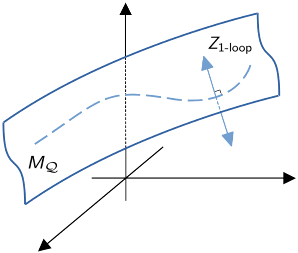
- Supercharge Q localizes observables to susy configurations

$$Z(\mathrm{SCFT}) = \int_{M_Q} d\phi \, \mu(\phi) \, e^{-S[\phi]} \, Z_{\text{1-loop}}(\phi)$$

- All non-susy modes are "paired up", only contributions from $Q\phi = 0$ modes.

- What remains is (often) a finite-dimensional manifold M_Q.

- This talk: exact CFT results give access to quantum corrections in gravity.



- Study 3d $\mathcal{N} \geq 2$ SCFTs decribing the low-energy limit of N M2 branes probing a conical singularity

Chern-Simons-matter (CSm) theories

dual to

M-theory on $\text{AdS}_4 \times X_7$

Choice of Sasaki-Einstein $X_7 \to$ CSm

$X_7 = S^7/\mathbb{Z}_k$ (free) $\to$ ABJM

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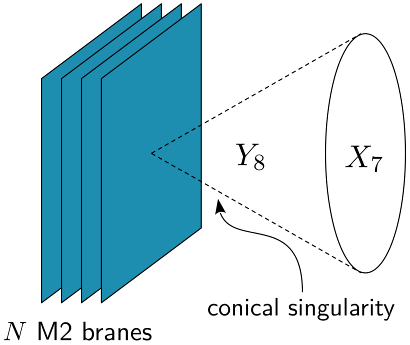
$X_7 = N^{010}, V^{52}, Q^{111}, \ldots$



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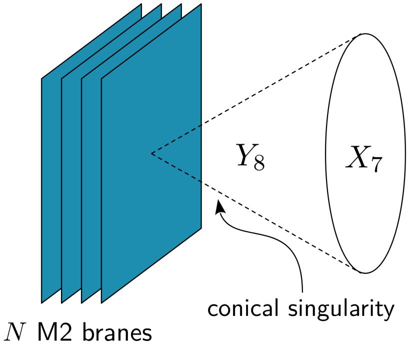
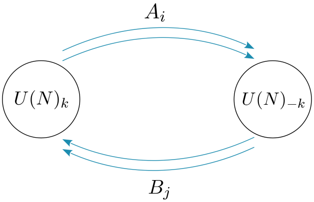


- Focus on ABJM theory for concreteness (ask me about other SCFTs).

The ABJM partition function

- ABJM is a 3d Chern-Simons-matter theory with high ($\mathcal{N} = 6$) supersymmetry.
  [Aharony,Bergman,Jafferis,Maldacena'08]



- Matter with quartic superpotential

$$W = \frac{4\pi}{k} \mathrm{Tr}[A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1]$$

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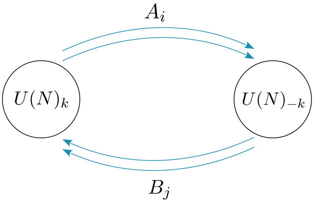

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- Exact partition function on S^3 via localization [Kapustin,Willett,Yaakov'09]

$$Z(S^3) = \frac{1}{(N!)^2} \int \frac{d^N\mu}{(2\pi)^N} \frac{d^N\nu}{(2\pi)^N} \exp\Big[\frac{ik}{4\pi} \sum_{i=1}^{N} (\mu_i^2 - \nu_i^2)\Big] Z_{\text{1-loop}}$$

$$Z_{\text{1-loop}} = \frac{\prod_{i<j} \left(2\sinh\left(\frac{\mu_i - \mu_j}{2}\right)\right)^2 \left(2\sinh\left(\frac{\nu_i - \nu_j}{2}\right)\right)^2}{\prod_{i,j} \left(2\cosh\left(\frac{\mu_i - \nu_j}{2}\right)\right)^2}$$

  1-loop factor includes vector multiplets & matter multiplets.

- Consider the ‘t Hooft limit where $N \to \infty$ and $\lambda = N/k \sim g\,N$ fixed.
- At small λ, the theory is (roughly) 2 independent CS theories. Saddle-pt:

$$\mathcal{F}_0(\lambda) = N^2\Big[\log(2\pi\lambda) - \frac{3}{2} - 2\log 2 + \mathcal{O}(\lambda^2)\Big]$$

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$\to$ the planar free energy "counts" the N^2 dofs of the $U(N)$ nodes.

- At large λ, the theory is dual to classical gravity on AdS_4 with $\partial(\mathrm{AdS}_4) = S^3$
- GR predicts the leading behavior

$$\mathcal{F}_0(\lambda) = N^2\Big[-\frac{\pi\sqrt{2}}{3\sqrt{\lambda}} + \mathcal{O}(\lambda^{-3/2})\Big]$$

$\to$ at fixed k and large N we expect a cancellation of the leading order eigenvalue distribution from N^2 down to $\sqrt{k}N^{3/2}$.

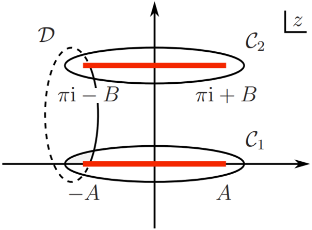
- Can we compute the exact interpolating function?

- From saddle-pt, introduce the resolvent $\omega(z) = \omega^{(\mu)}(z) + \omega^{(\nu)}(z + i\pi)$

$$\omega^{(\kappa)}(z) = \frac{2\pi i}{k} \left\langle \sum_{i=1}^{N} \coth\left(\frac{z - \kappa_i}{2}\right) \right\rangle$$

- As $N \to \infty$ eigenvalues condense on two cuts and planar resolvent $\omega_0(z)$ fixed by analyticity.

[Brézin,Itzykson,Parisi,Zuber‘78;...]



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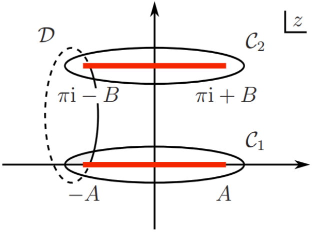

[Drukker,Mariño,Putrov'10]

- Planar free energy as a period integral

$$\partial_\lambda \mathcal{F}_0(\lambda) = \int_{\mathcal{D}} \omega_0(z)dz$$

$\to$ exact result in terms of hypergeometric & Meijer functions.

- Recovers weak coupling and reveals the full strong coupling expansion

$$\mathcal{F}_0(\lambda) = N^2\Big[-\frac{\pi\sqrt{2}}{3}\,\lambda^{-2}\Big(\lambda - \frac{1}{24}\Big)^{3/2} - \frac{\zeta(3)}{8\pi^2}\,\lambda^{-2} + \mathcal{O}(e^{-\sqrt{\lambda}})\Big]$$

- Going beyond the planar limit is challenging.
- Two methods:
  1. ABJM on $S^3 \leftrightarrow$ CS on $S^3/\mathbb{Z}_2 \leftrightarrow$ Topological String on local $\mathbb{P}^1 \times \mathbb{P}^1$ then use direct integration of the holomorphic anomaly equation
     [Drukker,Mariño,Putrov'10; Fuji,Hirano,Moriyama'11]
  2. Rewrite $Z(S^3)$ as a free Fermi gas of N particles with density matrix ρ

$$Z(S^3) = \frac{1}{N!} \sum_{\sigma \in S_N} (-1)^{\text{sig}(\sigma)} \int d^N y \prod_{i=1}^{N} \rho(y_i, y_{\sigma(i)})$$

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- Both 1. & 2. give the full resummation at large N and fixed k

$$Z(S^3) = \left(\frac{2}{\pi^2 k}\right)^{-1/3} e^{\mathcal{A}(k)} \,\mathrm{Ai}\left[\left(\frac{2}{\pi^2 k}\right)^{-1/3}\left(N - \frac{1}{3k} - \frac{k}{24}\right)\right] + \mathcal{O}(e^{-\sqrt{N}})$$

The holographic vantage point

- How does gravity see the CFT result? Expand the free energy at large N

$$\mathcal{F}(S^3) = -\frac{\pi\sqrt{2k}}{3}N^{3/2} + \frac{\pi(k^2+8)}{24\sqrt{2k}}N^{1/2} - \frac{1}{4}\log N + \ldots + \mathcal{O}(e^{-\sqrt{N}})$$

- The LO term is recovered from classical gravity on AdS.

[Emparan,Johnson,Myers‘99]

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- Start with two-derivative Einstein-Maxwell action

$$S_0 = -\frac{1}{16\pi G_N}\int_{M_4} d^4x\sqrt{g}\Big[R + 6L^{-2} - \frac{1}{4}F^2\Big]$$

- For M_4 asymptotically AdS, add counter-terms

$$S_{0,\mathrm{CT}} = -\frac{1}{8\pi G_N}\int_{\partial M_4} d^3x\sqrt{h}\Big[K - 2L^{-1} - \frac{L}{2}R_{(3)}\Big]$$

- Evaluate on-shell for AdS_4 with S^3 boundary and translate to CFT variables

$$\mathcal{I}_0 = \frac{\pi L^2}{2G_N} = \frac{\pi\sqrt{2k}}{3}N^{3/2}$$

- How do we get the NLO correction?
- Higher-derivative corrections with couplings $\lambda_{1,2}$

$$S = S_0 + \frac{\lambda_1}{32\pi^2}\, S_{\mathrm{W}} + \frac{\lambda_2}{32\pi^2}\, S_{\mathrm{GB}}$$

- Thanks to susy, only two HD Lagrangians

$$\begin{aligned}\mathcal{L}_{\mathrm{W}} &= W^2 - L^{-2}F^2 + \tfrac{1}{2}(F^+)^2(F^-)^2 - 4F^-_{\mu\nu}R^{\mu\rho}F_\rho^{+\nu} \\ &\quad + 8(\nabla^\mu F^-_{\mu\nu})(\nabla^\rho F_\rho^{+\nu}) + \tfrac{1}{6}(R + 12L^{-2})^2 \\ \mathcal{L}_{\mathrm{GB}} &= (R_{\mu\nu\rho\sigma})^2 - 4R_{\mu\nu}R^{\mu\nu} + R^2\end{aligned}$$

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- Must also include appropriate counter-terms.
- Evaluating on AdS_4 yields

$$\mathcal{I} = \left[\frac{\pi L^2}{2G_N} - \lambda_1\right] + (\lambda_1 + \lambda_2)\,\chi(\mathrm{AdS}_4) = \frac{\pi L^2}{2G_N} + \lambda_2$$

- Two effects compete at $\mathcal{O}(N^{1/2})$: HD and dictionary corrections.

Adding deformations

- In the CFT, turn on real masses $m_{1,2,3}$ in the Cartan of the flavor group.
- Convenient to use linear combinations

$$\Delta_1 = \frac{1 - i(m_1 + m_2 + m_3)}{2}\,, \quad \Delta_2 = \frac{1 - i(m_1 - m_2 - m_3)}{2}$$
$$\Delta_3 = \frac{1 + i(m_1 + m_2 - m_3)}{2}\,, \quad \Delta_4 = \frac{1 + i(m_1 - m_2 + m_3)}{2}$$

with linear constraint $\sum_a \Delta_a = 2$

- Localization gives a finite-dimensional integral, but now

$$Z_{\text{1-loop}} = \text{VMs} \times \prod_{i,j} s\Big(\Delta_1 + \frac{\mu_i - \nu_j}{2\pi}\Big)\, s\Big(\Delta_2 + \frac{\mu_i - \nu_j}{2\pi}\Big)\, s\Big(\Delta_3 - \frac{\mu_i - \nu_j}{2\pi}\Big)\, s\Big(\Delta_4 - \frac{\mu_i - \nu_j}{2\pi}\Big)$$

where $s(z) = \prod_{m,n=0}^{\infty} \frac{m+n+1-iz}{m+n+1+iz}$

- Now have a full function $Z(\Delta_a)$ to compute.

- No topological string or Fermi gas approach available for generic Δ.
- Can attack the problem numerically:
  1. at fixed k [Bobev,de Smet,Hong,VR,Zhang;WIP]
  2. in the 't Hooft limit with fixed $\lambda = N/k$ [Geukens,Hong'24]
- Analytic results available for "special" Δ-configurations. [Nosaka'15]

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- Analytic results available for "special" Δ-configurations. [Nosaka'15]
- All results point to the Airy form

$$Z(S^3;\Delta) = e^{\mathcal{A}(k,\Delta)} C_\Delta^{-\frac{1}{3}} \operatorname{Ai}\left[C_\Delta^{-\frac{1}{3}}(N - B_\Delta)\right] + \mathcal{O}(e^{-\sqrt{N}})$$

$$C_\Delta = \frac{1}{8\pi^2 k}\frac{1}{\prod_a \Delta_a}\,, \quad B_\Delta = \frac{k}{24} - \frac{\sum_a \Delta_a^{-1}}{12k} + \frac{1 - \frac{1}{4}\sum_a \Delta_a^2}{12k\prod_a \Delta_a}$$

- Reduces to known case when $\Delta_a = \frac{1}{2}$ (i.e. $m_i = 0$).
- Compatible with exchange symmetries of the Δ_a in the localized integral.

- Expand the Airy conjecture at large N

$$\mathcal{F}(S^3;\Delta) = -\frac{\pi\sqrt{2k}(16N^{3/2} - kN^{1/2})}{12}\sqrt{\prod \Delta_a} - \frac{2\pi}{3\sqrt{2k}}N^{1/2}F^{(0)}(\Delta) + \ldots$$

$$F^{(0)}(\Delta) = \frac{(\sum \Delta_a)^2 - \sum \Delta_a^2}{8\sqrt{\prod \Delta_a}} - \sqrt{\prod \Delta_a}\,\frac{\sum \Delta_a^{-1}}{\sum \Delta_a}$$

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- ... but the conjecture goes further: resum the $1/N$ corrections

$$\mathcal{F}(S^3;\Delta) = -\frac{4\pi\sqrt{2k}}{3}\left(N - \frac{k}{24}\right)^{3/2}\sqrt{\prod \Delta_a} - \frac{2\pi}{3\sqrt{2k}}\left(N - \frac{k}{24}\right)^{1/2}F^{(0)}(\Delta)$$

$$+ \text{lower homogeneity degree} + N\text{-independent} + \mathcal{O}(e^{-\sqrt{N}})$$

- Different organizing principle, based on homogeneity degree in Δ.

- On the gravity side, mass deformation $\rightarrow$ scalar fields z_a [Freedman,Pufu‘13]
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- Evaluate the gravity action on AdS_4 while keeping the VMs off-shell

$$\mathcal{I} = \frac{2\pi L^2}{G_N}\sqrt{\prod z_a} + \text{lower homogeneity degree in } z_a$$

- Gives the quantum corrections to dictionary

$$\frac{L^2}{G_N} = \frac{2\sqrt{2k}}{3}\left(N - \frac{k}{24}\right)^{3/2} = \frac{2\sqrt{2k}}{3}\left(N^{3/2} - \frac{k}{16}N^{1/2} + \dots\right)$$

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- and the value of the four-derivative coupling λ_2 in the M-theory EFT.
- In fact, mass-deformed CFT encodes the full z-dependence.

- Because of susy, the complete action of VMs coupled to gravity is controlled by a single function: the prepotential

$$F(z) = \sum_{p,q=0}^{\infty} F^{(2-2p-2q)}(z)\mathrm{GB}^p W^{2q}$$

- CFT localization computes this prepotential, e.g. for $(p,q) = (1,0)$

$$F^{(0)}(z) = \frac{(\sum z_a)^2 - \sum z_a^2}{8\sqrt{\prod z_a}} - \sqrt{\prod z_a}\,\frac{\sum z_a^{-1}}{\sum z_a}$$

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- Reminiscent of ungauged supergravity $(L \to \infty)$ where string amplitudes at genus g give rise to

$$\sum_g F_g(z) W^{2g}$$

couplings in the 4d effective action. [Antoniadis,Gava,Narain,Taylor'93]

- F_g computed by topological strings on CY_3 [Bershadsky,Cecotti,Ooguri,Vafa'93]

Type IIA string amplitudes

- Access the IIA regime using that metric on $S^7/\mathbb{Z}_k$ is a fibration over $\mathbb{CP}^3$

$$ds^2_{S^7/\mathbb{Z}_k} = \frac{1}{k^2}(d\varphi + k\omega)^2 + ds^2_{\mathbb{CP}^3}$$

  $d\omega$ is the Kähler form on the base.

- Reducing along the fiber gives the $\mathrm{AdS}_4 \times \mathbb{CP}^3$ background of Type IIA.

- The string coupling is

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- Perturbative string theory in the limit $N \to \infty$ and $\lambda = N/k$ fixed.
- Organize the large N CFT result à la 't Hooft

$$\mathcal{F}(S^3) = \sum_{g\geq 0}\left(\frac{2\pi i\lambda}{N}\right)^{2g-2}\mathcal{F}_g(\lambda)$$

- Corresponds to the genus expansion of the free energy on IIA side.

- Without mass deformations, use analytic Airy of M-theory to obtain

$$
\begin{aligned}
\mathcal{F}_0(\lambda) &= \frac{4\pi^3\sqrt{2}}{3}\,\hat{\lambda}^{3/2} + \frac{\zeta(3)}{2} \\
\mathcal{F}_1(\lambda) &= \frac{\pi}{3\sqrt{2}}\,\hat{\lambda}^{1/2} - \frac{1}{4}\log\hat{\lambda} + \frac{1}{6}\log\lambda + 2\zeta'(-1) - \frac{3}{4}\log 2 + \frac{1}{6}\log\frac{\pi}{2} \\
\mathcal{F}_2(\lambda) &= \frac{5}{96\pi^3\sqrt{2}}\,\hat{\lambda}^{-3/2} - \frac{1}{48\pi^2}\,\hat{\lambda}^{-1} + \frac{1}{144\pi\sqrt{2}}\,\hat{\lambda}^{-1/2} - \frac{1}{360} \\
\mathcal{F}_3(\lambda) &= \frac{5}{512\pi^6}\,\hat{\lambda}^{-3} - \frac{5}{768\pi^5\sqrt{2}}\,\hat{\lambda}^{-5/2} + \frac{1}{1152\pi^4}\,\hat{\lambda}^{-2} \\
&\quad - \frac{1}{10368\pi^3\sqrt{2}}\,\hat{\lambda}^{-3/2} - \frac{1}{22680} \\
&\dots
\end{aligned}
$$

$$
\hat{\lambda} = \lambda - \frac{1}{24}
$$

- genus-g free energies of Type IIA on $\mathrm{AdS}_4 \times \mathbb{CP}^3$ up to $\mathcal{O}(e^{-\sqrt{\lambda}})$

- Duality ABJM on $S^3 \leftrightarrow$ Topological String on local $\mathbb{P}^1 \times \mathbb{P}^1$.
- Useful for non-perturbative free energies. At $g = 1$

  [Huang,Klemm'06; Mariño,Pasquetti,Putrov'09; Drukker,Mariño,Putrov'11]

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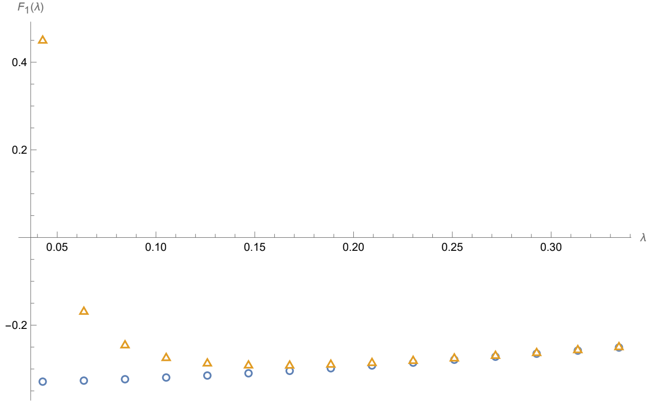
- By holomorphic anomaly equation, higher genus take the form

$$\mathcal{F}_{g\geq 2}(\lambda) = \frac{1}{(bd^2)^{g-1}} \sum_{n=0}^{3g-3} E_2^n(\tau)\, p_n^{(g)}(b,d)$$

  $b = \vartheta_2^4(\tau)$, $d = \vartheta_4^4(\tau)$ and p_n are polynomials in b and d.

- The p_n can be found algorithmically, but no known generating functional.

- Compare to our perturbative results. At genus 1,



Triangles: perturbative result; circles: topological string.

- Easy to extract higher genus from IIA expansion of Airy.
  More efficient than solving the holomorphic anomaly equation.

- With mass deformations, no duality to TS.
- Extract perturbative free energies from Airy conjecture.
- Setting $m_3 = 0$, $m_\pm = m_2 \pm m_1$ for illustration

$$\mathcal{F}_0(\lambda; m_\pm) = \frac{4\pi^3\sqrt{2}}{3}\sqrt{(1+m_+^2)(1+m_-^2)}\,\hat{\lambda}^{3/2} + \frac{\zeta(3)}{2}\,\frac{2-m_+^2-m_-^2}{2}$$
$$\ldots$$

up to $\mathcal{O}(e^{-\sqrt{\lambda}})$ corrections.

- Can such results be obtained directly in IIA strings?
  Need to consider AdS backgrounds with running scalars...

Further corrections

- Focus so far on LO and NLO terms in the large N expansion

$$\mathcal{F}(S^3) = -\frac{\pi\sqrt{2k}}{3}N^{3/2} + \frac{\pi(k^2+8)}{24\sqrt{2k}}N^{1/2} - \frac{1}{4}\log N + \ldots + \mathcal{O}(e^{-\sqrt{N}})$$

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- The log term comes from one-loop effects around the AdS_4 background of the Kaluza-Klein gravity theory after reduction on $S^7/\mathbb{Z}_k$.

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- Must deal with infinitely many fields in the "EFT".

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- Explicit precision checks of AdS/CFT involving quantum gravity effects.

Conclusion

- Localization in SCFTs encode quantum gravity effects on AdS.
- Can also study $S^1 \times S^2$ topologies rather than S^3
  $\rightarrow$ relevant for BPS black holes in AdS.

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Thank you for your attention!